The Auditor and Statistical Sampling Techniques

Marilyn Metcalf
Ouachita Baptist University

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THE AUDITOR
AND
STATISTICAL SAMPLING TECHNIQUES

Honors 482

Presented to Associate Professor Margaret Wright of the
Ouachita Baptist University Faculty
In Fulfillment of the
Requirements

For

Honors Special Studies

By

Marilyn Metcalf
Arkadelphia, Arkansas
Fall 1973
"The auditor's overall objective is to determine whether the accounting process is free from material error."¹ To complete his objective, the auditor must thoroughly analyze all aspects of the financial statements used in the accounting process in order to satisfy himself that they fairly represent the results of activity for the period under audit and of the financial position at the end of that period. If the volume of the transactions is great, time and economic factors may keep the auditor from making a 100 percent examination. Since a complete examination cannot be made, the auditor must choose a portion of the transactions to analyze. When the auditor has to do this he has a sampling problem. From this portion he must make inferences about all of the data.

In auditing two different types of sampling are used—judgment sampling and statistical sampling.

Judgment sampling is the selection of items to be tested on a subjective basis. Items are chosen without consideration of the statistical requirements for sample size, method of selection, or evaluation of results. The selection criteria are based on the subjective judgment of the auditor. In those situations where the auditor does not need

to project from the sample results to the entire population judgment sampling is appropriate. The use of judgment sampling is limited because there is no mathematical basis for projecting the sample results to the total population. ²

Judgment sampling was the traditional method used by the auditor. Statistical sampling is the same general process as the judgment sampling but involves certain refinements. The method requires the auditor to define clearly the standards used in arriving at the sample size, and it results in a sample which is representative of the entire group. The auditor must use mathematical techniques to do this.

Statistical sampling is not a substitute for the auditor's judgment, but instead gives him additional information upon which to base his opinions about the financial statements. The independent auditor must still rely on his own judgment to determine the extent of the audit sampling.

"In a special report entitled 'Statistical Sampling and the Independent Auditor,' the Committee on Statistical Sampling of the AICPA concluded as follows:

The committee is of the opinion that the use of statistical sampling is permitted under generally accepted auditing standards. Such standards have for some time recognized the acceptability and effectiveness of testing. In this context, statistical sampling simply furnishes a means of selecting items for some audit tests and, where

². Ibid., p. 167.
the auditor thinks information desirable, a means for describing mathematically some of the results of the test. In other words, statistical sampling may furnish some assistance in testing, both in selecting items and in evaluating the results of the test.

While the committee concludes that statistical sampling techniques are permitted under generally accepted auditing standards, the choice of testing methods is left to the judgment of the auditor. In exercising this choice, the auditor needs to study statistical sampling sufficiently to be able to recognize areas in which this technique will improve his auditing procedures and achieve audit economies yet retain the usual degree of assurance."

Statistical sampling does provide an objective and defensible means to the auditor of using his professional judgment in a desirable manner. He can objectively defend his conclusions about the population to his supervisor, the client, or to outside parties if questions arise about the sample. Also, this method often allows him to attain a satisfactory level of assurance about a population at less cost.

"The application of any statistical sampling plan has five distinct phases in which the auditor must---

1. Define the objectives and nature of the test.
2. Determine the method of sampling to be used.

3. Calculate the sample size.
4. Select the sample.
5. Evaluate the sample.

This entire process is described as statistical sampling."

"Generally, statistical sampling may be separated into
two distinct phases: statistical selection and statistical
measurement. Statistical selection refers to the method of
drawing the sample, whereas statistical measurement involves
the calculation of the sample size and the evaluation of
sample results. Statistical selection may be used independ­
dently from statistical measurement."

**Statistical Selection**

The auditor may use four statistical methods of selecting
items for examination: random number table selection, systema­
tic selection, stratified selection, and cluster selection.
Each of these methods permits the auditor to select a sample
from the population in such a manner that each item in the
group has the same chance of being chosen for examination as
every other item. The general process is known statistically
as "random sampling" and is sometimes described as "probability
sampling." (Random sampling relates only to selecting the
sample, therefore, is only part of the statistical sampling
procedure and not the entire process.)

4. Ibid.
5. Ibid.
I. Random Number Table Selection

Random number tables are perhaps the easiest method of selecting items on a random basis. Each digit on the table is a random digit and is arranged in columnar form so the reader can select the number more easily. The auditor may put the numbers in any columnar arrangement he chooses and may start at any point he wishes.

It is not easy to produce a truly random set of numbers so the auditor should pick a table that has been thoroughly tested for randomness. After he has done this he may proceed with his selections. The auditor should decide which route or path he wishes to take and remain consistent throughout his selections. The initial digits are usually selected at random and the remaining digits should be selected by a pre-arranged formula. The numbers selected from the table must correspond with the sequence of numbers in the group of items being examined. If a number appears on the table which is not within the sequence of transactions being examined the auditor skips the number. For greater efficiency in situations where parts of the identifying numbers of the items to be sampled are the same throughout the group, the similar parts should be ignored.

Sometimes the items to be examined are either numbered in blocks with the first digit, or digits, representing
**Figure 1**
RANDOM NUMBER TABLE

<table>
<thead>
<tr>
<th>Column</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>1</td>
<td>10480</td>
<td>15011</td>
<td>01536</td>
<td>02011</td>
<td>81647</td>
<td>91646</td>
</tr>
<tr>
<td>2</td>
<td>22368</td>
<td>46573</td>
<td>25595</td>
<td>85393</td>
<td>30995</td>
<td>89198</td>
</tr>
<tr>
<td>3</td>
<td>24130</td>
<td>48360</td>
<td>22527</td>
<td>97265</td>
<td>76393</td>
<td>64809</td>
</tr>
<tr>
<td>4</td>
<td>42167</td>
<td>93093</td>
<td>06243</td>
<td>61680</td>
<td>07856</td>
<td>16376</td>
</tr>
<tr>
<td>5</td>
<td>37570</td>
<td>39975</td>
<td>81837</td>
<td>16656</td>
<td>06121</td>
<td>91782</td>
</tr>
<tr>
<td>6</td>
<td>77921</td>
<td>06907</td>
<td>11008</td>
<td>42751</td>
<td>27756</td>
<td>53498</td>
</tr>
<tr>
<td>7</td>
<td>99562</td>
<td>72905</td>
<td>56420</td>
<td>69994</td>
<td>98872</td>
<td>31016</td>
</tr>
<tr>
<td>8</td>
<td>96301</td>
<td>91977</td>
<td>05463</td>
<td>07972</td>
<td>18876</td>
<td>20922</td>
</tr>
<tr>
<td>9</td>
<td>89579</td>
<td>14342</td>
<td>63661</td>
<td>10281</td>
<td>17453</td>
<td>18103</td>
</tr>
<tr>
<td>10</td>
<td>85475</td>
<td>36857</td>
<td>53342</td>
<td>53988</td>
<td>53060</td>
<td>59533</td>
</tr>
</tbody>
</table>

*Published by the Bureau of Transport Economics and Statistics of the Interstate Commerce Commission*

**Figure 2**
WORK PAPER FOR SELECTION OF RANDOM NUMBERS

**Main Series**

<table>
<thead>
<tr>
<th>Sub­-digits</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>01</td>
</tr>
<tr>
<td>10-19</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22</td>
<td>27</td>
</tr>
<tr>
<td>30-39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60-69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>80-89</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-99</td>
<td>91</td>
<td>99</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the month in which the transaction took place, or the transactions are numbered alphanumerically, or are unnumbered. In these cases the auditor may use two series of random numbers: the first series representing the month, the alphabetical designation or the page number, and the second series representing the transaction or line numbers.

In using the random number table, the auditor may occasionally find that he draws the same number more than once. Should this happen, he may either ignore the duplicate number or he may examine it more than once. If he ignores the duplicate number, he is said to be "sampling without replacement." If he examines it for a second or third or more times, he will be "sampling with replacement." Statistical theory would require him to sample with replacement; however, as a practical matter, it seems undesirable to examine the same transaction, more than once. Therefore, in the application of statistical sampling in auditing, the usual procedure is to ignore duplicate numbers and to "sample without replacement."

Since random number tables appear in book form, the auditor may prefer to record the numbers of the documents inspected on an adding machine tape or work paper rather than to mark in the book the items selected for examination. The use of an adding machine tape is inconvenient for the purpose of pulling the selected documents from the files, however, because the numbers listed on the adding machine tape will be in a different order than the documents located in the file.

Two more efficient methods are available. The auditor may record the numbers selected on cards and later sort these (or have the client's employees sort them) in numerical order. He may prefer to use a work paper such as that illustrated in Figure 2.
Thus, if he draws number 401, he merely writes "01" in the column headed 400 and so forth, thereby creating a work paper that shows in convenient orders all numbers selected. The work paper may be expanded when larger series of numbers are involved.

An obvious advantage of using random numbers is that no one, not even the auditor himself, can predict in advance what units from the population he will choose.

II. Systematic Selection

Auditors often use systematic selection to choose items from a group for examination. This method selects every \( n \)th unit of the population where \( n = \frac{p}{s} \), \( p \) being the population, and \( s \) being the required sample. The auditor must be careful to see that the interval \( n \) does not correspond to some pre-existing cyclical sequence in the population. If it did correspond closely to a group sequence most of the sample drawing might come from the same group and might not represent the population as a whole. To assure that a random sample will be drawn, the auditor should make sure that the population is arranged in a random sequence. Another factor to look out for is transactions of a particular type processed all at one time during each month or accounting period, possibly eliminating these items from the sample. A further guard against a nonrandom sample is to use two or more random starting points.

6. Ibid., p. 789.
The advantage to the auditor of using the systematic selection technique is being able to obtain a sample from a population of unnumbered documents or transactions. There is no need for the unnumbered documents to be numbered physically or mentally, as would have to be done if a random number table were used. The auditor just counts off the sampling interval or uses a ruler to measure the interval and select the documents.

III. Stratified Selection

There is no requirement that only one method of statistical sample selection be applied to the entire group of items to be examined. Rather, the population may be stratified and different techniques applied to each stratum, if desired. Stratified selection is not by itself a technique of drawing samples. However, it is useful for two purposes: to arrange the population into various strata of significance and to utilize different sample selection techniques. The auditor is usually concerned with the materiality of transactions, as well as with the type of transactions. To examine a population composed of items having varying dollar values, or which represent different types of transactions, the auditor may stratify it and apply different procedures to the various strata. For example, if the auditor wishes to use statistical sampling techniques to confirm accounts receivable, he might stratify and test as follows:

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Composition of Stratum</th>
<th>Method of Selection Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All accounts of $10,000 and over</td>
<td>100% examination</td>
</tr>
<tr>
<td>Stratum</td>
<td>Composition of Stratum</td>
<td>Method of Selection Used</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>2</td>
<td>Wholesale accounts receivable (under $10,000), all numbered with numbers ending in zero.</td>
<td>Random number table selection</td>
</tr>
<tr>
<td>3</td>
<td>All other accounts (under $10,000) in random order</td>
<td>Systematic selection</td>
</tr>
</tbody>
</table>

The above illustrates the stratification of a population not only by dollar values but also by transaction type. In addition to stratifying the population on these bases, the auditor may also stratify by transaction frequency. For instance, in a test of internal control, the auditor may wish to stratify the population into high and low volume transactions when he knows that the controls are more likely to be violated during the processing of low volume transactions.

Stratified sampling has at least two major advantages to the auditor. It enables the auditor to relate sample selection to the materiality, turnover, or other characteristics of items and to apply different auditing procedures to each stratum. The method is also favored by statisticians since it improves the reliability of the sample. Whenever items of extremely high or low values, or of unusual characteristics, are segregated into separate fields, the samples drawn will be more reliable.

IV. Cluster Sampling

If a sample, say, of 300 units is required, rather than selecting 300 random numbers and 300 independent units from the population, the sampler selects, say, 30 clusters of 10 units each. Since conventional audit procedures use cluster sampling, the cluster being one month, one week, one department, etc., it might be thought that cluster sampling provides a vehicle for drawing traditional and

7. Ibid., p. 792-793.
statistical audit procedures closer together. This is not so. The statistician is always careful to qualify his advocacy of cluster sampling with the remark that the minimum number of clusters must be of the order of 20. The number of clusters used in auditing is almost invariably very much less than this, usually less than half a dozen, and often a single cluster.

The trouble with cluster sampling is that the variability of the characteristics within a cluster is likely to be less than the variability between clusters. This is another way of saying that the sample drawn from a cluster will not provide a true reflection because they have something in common, the same date, customer, product, etc. This common factor may invalidate an inference about a population made from a sample drawn from a cluster. On the other hand it need not do so. If cluster sampling is particularly appropriate to a given audit situation, the auditor would be well advised in the first year of the audit to select several clusters and test the variability of the condition, i.e. error value, he is attempting to assess by comparing them with a random sample of equal size. If the variability within clusters is not significantly different from the variability between them or the pure random sample, he can with reasonable confidence use cluster sampling in future years.

**Statistical Measurement**

When an auditor uses statistical measurement techniques he will usually use statistical selection in drawing the sample to be analyzed but there may be times that he uses the judgment basis to determine the size of the sample.

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Four statistical measurement techniques are usually discussed in auditing literature. These are:

1. **Estimation sampling for attributes.** This method permits the auditor to determine the rate of occurrence of certain characteristics (for example, exceptions) in a population within prescribed ranges of precision and levels of confidence.

2. **Estimation sampling for variables.** This method permits the auditor to determine an aggregate dollar amount of accounting data within prescribed ranges of precision and level of confidence.

3. **Acceptance sampling.** This method permits the auditor to reject or accept a population in accordance with a stipulated allowable error rate at a predetermined confidence level.

4. **Discovery sampling.** This method permits the auditor to determine, with assurance at a prescribed confidence level, whether at least one deviation (exception) exists in a population.9

To understand statistical measurement techniques the auditor must understand the following concepts:

**Population:** refers to the group of items to be examined and is synonymous with universe or field.

**Sample:** refers to a group of items drawn from the population for examination on a random or "probability" basis. **Sample item** refers to one item of the group drawn for examination.

**Occurrence rate:** indicates the number of times a given characteristic occurs in the population being studied. Since the auditor in many of his tests is concerned with the number of exceptions or errors, he may often use the term **error rate** rather than occurrence rate.10

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Precision: In any sampling process, whether it be statistical or judgment, the auditor rarely has assurance that the results of the sample will be fully representative of the characteristics of the population. The possibility of sampling error (a sample which indicates characteristics that are not the actual characteristics of the population) is always present. In utilizing statistical measurement techniques, the auditor measures the possibility of sampling error by calculating precision and confidence level. In determining precision the auditor specifies the + limits from the sample results to indicate the range within which he expects the true value of the characteristics in the population to lie.11

(The range is referred to as the precision interval and the highest and lowest values of the range are the precision limits.)

\[
\text{Precision} = \text{Confidence coefficient} \times \text{Standard error of sample estimate at desired confidence level}
\]

Confidence Level: In addition to making a decision on precision, the auditor must decide upon the level of risk that the sample will not be representative of the population as a whole, within the range of precision stipulated. Confidence level is the measure of the number of times in 100 that the sample result, in the range of desired precision, will represent the results from a 100 percent examination of the population.

(The confidence level is greatly influenced by the effectiveness of the internal control system. As the effectiveness of the internal control increases, the confidence level % can be decreased.)

Precision is the range within which the answer may vary yet be acceptable; confidence level measures the likelihood that the answer will fall within that range.

Expected occurrence rate: To select an economical sample size under many sampling methods, the auditor

11. Ibid., p. 795.
must make an approximation of the expected occurrence rate or the maximum acceptable occurrence rate in addition to determining precision and confidence level. The determination of the expected occurrence rate is made after considering the type of transaction involved, the purpose of the test, the materiality of the items, the system of internal control, and the auditor's knowledge of the client's operations and business.12

**Maximum acceptable occurrence rate:** The most occurrences the auditor will accept. The maximum acceptable occurrence rate may be determined either by approximating the expected occurrence rate and adding to that the upper limit of precision, or it may be determined on a judgment basis.13

**Standard Error:** is a measure of sampling error.14

**Sampling Error:** A difference between the sample estimate and the true population characteristic that results solely because of the random incidence of population items in the sample. Sampling error is an inaccuracy in the sample estimate caused by inherent properties of the sampling method.

No sample error if sample size = size of population.15

**Standard Deviation:** The standard deviation is computed by determining the deviation of each item in the population from the mean, squaring these deviations, computing the sum of the squared deviations, dividing this sum by the number that is one less than the number of items in the population, and calculating the square root of the resulting quotient.16

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12. Ibid. p. 796.
13. Ibid., p. 797.
15. Ibid., p. 171.
The following diagram shows the relationships between the four sampling methods.

The basic division is between attributes and variables sampling. Acceptance sampling is a simplified form of estimation sampling of attributes, and discovery sampling is a simplified form of acceptance sampling where the acceptable number of defectives is zero.

Estimation sampling of attributes and its two simplified forms are used to estimate the proportion of a population having a given attribute, while estimation sampling of variables is used to measure the total value of some variable of a population.  

I. Estimation Sampling of Attributes

Estimation sampling for attributes is used by the auditor testing for compliance with internal control procedures. "A departure from an essential internal control procedure is a compliance error. This type of error is called an attribute by statisticians. An attribute is a quality characteristic of an item."  

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Attributes sampling measures the frequency with which a specified occurrence or attribute exists in the population. The most common type of attribute of interest in auditing is the frequency with which errors occur in the population being tested. Each item in the population either does or does not contain the attribute being measured. On the basis of the sample the auditor can then compute the percentage of the population that contains the attribute.

To assure himself that compliance does or does not exist, the auditor requires proof of performance. He usually finds this in source documents and ledger accounts. This evidence is known as documentary evidence.

"The following points provide an outline of the methodology for using attributes sampling:

1. State the objective of the audit test.
2. Define the population.
3. Decide on the appropriate upper precision limit.
4. Decide on the appropriate confidence level.
5. Make an advance estimate of the population occurrence rate.
6. Determine the proper sample size.
7. Randomly select the sample.
8. Perform the audit procedures.
9. Generalize from the sample to the population.
10. Decide on the acceptability of the population."

Most of the time attributes sampling is used in testing transactions to see if reliability on the system of internal

control can be established. Occasionally it is used to verify that an account balance has been fairly stated. Before the testing is started the auditor should carefully define the characteristics of an error so that staff members will be able to identify exceptions in the sample.

The auditor must define the population in advance. He may generalize from the data he selects but he must remember that he must randomly sample from the entire population as he has defined it. The auditor can do no more than generalize about the population on the basis of the sample. He cannot state the exact characteristics of the population because they remain unknown to him.

Setting the precision limits would be the auditor's next step. "Precision is the statistical measure of the range of values, less than or more than the sample results, within which the true population value is expected to fall. The lower and upper extremes of this range are known as the precision limits."^20

In attributes sampling the auditor is more interested in the upper precision limit because, if the true population error rate is large, the auditor will want to know about it, but if the true population error rate is small, the population is acceptable.

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20. Ibid., p. 165.
Even though the auditor sets precision limits, he still takes a risk that the sample results will not correctly reflect the true population characteristics. By using statistical sampling the auditor can measure the risk. In attributes testing the confidence level (also known as reliability) is the means of expressing the probability that the true population error rate is less than the upper precision limit. This is usually expressed as a percentage.

The auditor must use his best judgment based on his professional experience to set the precision limits and the confidence level. The upper precision limit will depend on the materiality in the particular case and the definition of error. The confidence level will largely depend on the reliability of internal control.

"In attributes sampling, an advance estimate of the population error rate is necessary to determine the appropriate sample size. If the estimate of the population error rate is small, the auditor will be able to limit himself to a relatively small sample size to satisfy his upper precision limits."21

Four factors determine the sample size the auditor should use in attributes sampling: the population size, the advance estimate of the population

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error rate, the upper precision limit believed by the auditor to be appropriate, and the reliability desired by the auditor. Once these factors have been determined, the auditor can calculate the necessary sample size by using one of the available attributes sampling tables. The CPA candidate should understand the effect on the sample of individually changing each of the four factors that determine its size when the other factors remain constant. The following table illustrates the effect on the sample of increasing each of the four factors:

<table>
<thead>
<tr>
<th>Change the Following Factor while the Three Other Factors are Held Constant</th>
<th>Effect on the Required Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Increase reliability</td>
<td>Increase</td>
</tr>
<tr>
<td>2. Increase upper precision limit</td>
<td>Decrease</td>
</tr>
<tr>
<td>3. Increase the advance estimate of the population error rate</td>
<td>Increase</td>
</tr>
<tr>
<td>4. Increase population size</td>
<td>Increase(^22)</td>
</tr>
</tbody>
</table>

After the auditor has computed the sample size he must choose the sample by random selection. The two most prevalent methods used are random number tables and systematic sampling. The auditor then performs the normal audit procedures on the items of the sample, keeping a record of all errors found.

The auditor computes the upper precision limit and the reliability for the population based on the actual sample results. Using this information he can generalize about the population. The auditor then refers back to the precision and confidence standards set prior to the sampling. The results computed on the basis of the actual sample results must be at

least as good as the standards that were set in advance of the actual tests. If they are, the auditor can accept the results and issue an unqualified opinion on the basis of them.

If the reliability and upper precision limit standards of the sample do not satisfy the predetermined standards the auditor can increase his sample size to the point of certainty that the actual population error rate is acceptable or unacceptable, or he can request the client to take corrective action on the population. It is important that the standards initially set be fulfilled or the auditor will have to issue a qualified opinion.

II. Estimation Sampling of Variables

The auditor will use estimation sampling for variables when he is testing for material misstatement in dollar amounts. Since the main goal of auditing is to determine whether account balances are fairly stated on the financial statements, the use of variables sampling is often considered more directly applicable to auditing than attributes sampling.

An error that directly effects the dollar amount of a financial statement item is a monetary error. Monetary amounts are quantity characteristics of data. In statistical terms, quantity characteristics are referred to as variables. Although there are many causes of monetary errors, actual errors will always be in the form of either overstatements or understatements of balances.
Monetary errors may be analyzed in two ways when using estimation sampling. The total dollar value of an item may be estimated and then compared with the book value as a test of reasonableness. On the other hand, the amount of monetary error may be estimated directly by a difference estimate computed by comparing audited and book values on an item-by-item basis.23

In variables sampling as in attributes sampling, there are four major factors to consider in determining the sample size. These are the population size, the acceptable reliability level, precision, and the standard deviation of the population.

The population size is defined as the total number of elements in the population being tested. The reliability or confidence level is a measure of the assurance that the true dollar value of the population neither exceeds the upper precision limit or falls below the lower precision limit. This confidence level is decided by the auditor using his professional judgment.

The upper and lower precision limits must be computed. This is similar to attributes sampling, but it is stated in dollar amounts rather than as the percentage of errors in the population. In variables sampling the auditor is interested in both the upper and lower precision limits, whereas with attributes sampling he was concerned only with the upper limit.

The reason for this is that the auditor must determine whether the correct dollar value of an account is probably materially greater or less than the recorded value, but he need not be concerned if the true error rate in the population value is extremely small.

The fourth item which the auditor must be concerned about is the population standard deviation. This can probably be best understood by explaining the theory underlying estimation sampling.

The distribution of a population of individual values in relation to the arithmetic mean of the population constitutes the underlying basis for estimation sampling. It is known, for example, that 68.3 percent of the items in a normally distributed population will occur within plus or minus one standard deviation from the mean, 95.4 percent within plus or minus two standard deviations, and 99.7 percent within plus or minus three standard deviations.

A second important statistical concept involved in estimation sampling describes the results of taking a series of samples of a stated size and calculating the mean of the items in each sample. The means of the samples will tend to form a normal distribution, with the mean of the sample means equal to the mean of the population. The standard deviation of the sample means (the standard error) also is important, and particularly significant is the behavior of the standard deviation of the sample means in relation to the size of the samples. The larger the sample size from which the sample means are computed, the smaller will be the standard deviation of the sample means. This relationship of sample size and the standard deviation of the sample means is indicated by the equation:

\[ \text{Standard Error} = \frac{\sigma}{\sqrt{n}} \]

24. Steller, _loc.cit._
Standard deviation = \text{Standard deviation of population of sample means} = \frac{\text{Standard deviation of population of sample means}}{\sqrt{\text{Sample size}}}

The appropriate sample size can be determined either by the use of a formula or by using variables sampling tables once the population size and standard deviation have been determined.

As in attributes sampling, it is also important in variables sampling to look at the effect on the sample of individually changing each of the four factors that determine the sample's size. The relationships are as follows:

<table>
<thead>
<tr>
<th>Change the Following Factor while the Three Other Factors are Held Constant</th>
<th>Effect on the Required Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Decrease reliability</td>
<td>Decrease</td>
</tr>
<tr>
<td>2. Decrease precision level</td>
<td>Increase</td>
</tr>
<tr>
<td>3. Decrease the population standard deviation.</td>
<td>Decrease</td>
</tr>
<tr>
<td>4. Decrease the population size</td>
<td>Decrease$^2$</td>
</tr>
</tbody>
</table>

The auditor's final step will be to accept or reject the recorded value of the account being tested. If the values of the computed upper and lower precision limits are both between the upper and lower precision limits required by the auditor, the population is considered acceptable. If the population is not satisfactory, the sample size can be increased until the auditor knows whether the population value is acceptable or unacceptable, or the client can be requested to take corrective action.

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III. Acceptance Sampling

The auditor's job is to state whether the financial statements of a business are a fair representation of the accounting process and business assets. An excessive amount of error would make the financial statement figures unacceptable. If there are errors they are most often found in the client's inventory counts, inventory prices and extensions, accounts receivable aging, the recording of employees' hours worked in payroll calculations, or the classification of expenses in the voucher register. Acceptance sampling is helpful to the auditor in deciding whether errors are present or not and upon this decision whether the financial records are acceptable or not.

The theory of probability as it relates to acceptance sampling is perhaps best explained through the use of the classical illustration of a bowl containing white balls and black balls. For our purposes, assume that the bowl contains 10,000 balls, of which 9,000 are white and 1,000 are black, or in terms of percentages, 90 per cent of the balls are white and 10 per cent are black.

A random sample of the contents of the bowl can be taken by drawing one ball from the bowl, recording its color, replacing the ball, stirring the contents, and repeating the process until a desired number of balls has been drawn. If the process is stopped after 100 draws, it is theoretically possible that all 100 balls selected would be white, or that all 100 balls selected would be black. As a matter of fact, however, we would expect the tabulation of our selection to show that the distribution of black balls and white balls approximates more nearly the distribution of the full 10,000 balls in the bowl.
In other words, the sample would probably show 90 white balls and 10 black balls, or a distribution varying only slightly from this 90-10 relationship.

If 100 balls were to be drawn as described above, and that sampling process were to be repeated 1,000 times, the statistician could compute in advance the likely overall results of the thousand samples. This type of computed forecast is based upon probability.26

Before the auditor begins the sampling he must decide for himself how many errors he can allow and still retain the needed percentage of confidence or reliability in the population. This would be setting a precision limit and a confidence level. After he has done this he can proceed with the sampling. The sample results will not ascertain the exact percentage of error but the auditor will be in a better position to infer what the percentage is not.

If in the above example as many as 10 per cent of the balls in the bowl were actually black, you could expect to find a sample of 100 containing 2 black balls or less only about 2 times in 1000. Because this would be an extremely rare occurrence, the auditor can infer with considerable confidence that the black balls in the bowl do not exceed 10 per cent of the total, and on this basis accept the bowl of balls as meeting the minimum standard that had been set.

The auditor must now take his sampling results and make a "yes" or "no" decision as whether to accept the population or not. If he should decide to reject it, his alternatives would be to conduct a 100 percent examination or to qualify the audit report. Another factor to remember concerning acceptance sampling is that it provides for a lessor degree of risk in rejecting an acceptable population than in accepting a population that should be rejected. The result is the possibility that populations containing rates of error less than the specified acceptable level will be rejected.

IV. Discovery Sampling

Discovery sampling is a special case of attributes sampling. It is concerned with determining the rate of error in the population. Discovery samples try to encounter one erroneous item if there are erroneous items in the population. "The sampling plan is useful, however, only if erroneous or fraudulent items occur in the population with a frequency in excess of about 1 per cent, for no sampling plan can provide meaningful assurance of discovering a 'rare' item in a population. The only solution to the detection of the rare item is 100 per cent inspection." 27 Discovery sampling is best suited to the attempt to discover the existence of fraud when fraud is suspected.

27. Steller, Ibid., p. 547.
The definitions of the error, the expected error rate, and the upper precision limits are the only differences between discovery sampling and attributes sampling. In discovery sampling, an error is so defined that its occurrence will be seriously regarded. The expected error rate for discovery sampling is always zero since the auditor would normally not expect to find any of these critical errors in the sample.

As in attributes sampling the auditor must ahead of time decide what constitutes an error and define the population. He must determine the population size, the upper precision limit, and the desired reliability or confidence level. After this has been done the auditor randomly selects his sample, and performs the audit procedures to determine if there are errors in the sample. After the tests the auditor evaluates the results. If he finds no errors he can say with his predetermined level of confidence that the error rate within the population is within the limits of precision or tolerability.

If the auditor discovers an error in the sample he knows that the upper precision limit at a given confidence level exceeds his initial standards. The auditor in this case will usually switch to an attributes sampling table.

Sometimes during audit tests auditors will use attributes and discovery sampling simultaneously. Discovery sampling
will be used to evaluate the population for critical errors and attributes sampling is used for evaluating less important errors.

Considerations in the Use of Statistical Sampling in Auditing

With the increasing information concerning statistical sampling techniques the auditor will want to look at how they can be of use to him in the future. The following are considerations in the use of statistical sampling in auditing at the present time:

1. Statistical sampling techniques are consistent with generally accepted auditing standards, which recognize the appropriateness of sampling.

2. In a given audit, statistical techniques may be deemed to be appropriate in one or more areas of the audit but not in others, and the use of such techniques in one area of an audit imposes no responsibility for using them in other areas where they are not appropriate.

3. If the techniques are used in one area of an audit, the auditor would be well advised to be prepared to defend the nonuse of statistical techniques in other areas of the audit where sampling is involved.

4. The use of any one statistical technique in an audit, as for example random selection, does not in itself require the use of other techniques, such as statistical determination of sample size or statistical evaluation of the sample results.

5. Statistical techniques appear to be most useful when the auditor is dealing with voluminous data and his tests of the data are not closely related to other areas of the audit. At the present time, known statistical techniques do not offer a satisfactory means of establishing a quantitative
relationship between the results of a given audit test and other tests, or audit procedures that are closely related to it.

6. Improved control over audit procedures and more precise evaluation of sampling results are gained when objective statistical techniques are substituted for subjective judgmental selection and evaluation.

7. Knowledge of statistical techniques by auditors appears desirable so that the benefits of the techniques may be realized where the techniques are applicable, but the auditor should not have to become a trained statistician to obtain such knowledge.

8. If the auditor has only a general knowledge of statistics, he should seek the consultation and assistance of a trained statistician in resolving difficult applications of statistical techniques.

9. Statement on Auditing Procedure No. 36 points out that, contrary to the generally held assumption that "Extensions of Auditing Procedure" requires a client to make a complete physical count of all inventory items each year, a sufficiently accurate determination of inventories may be made without an annual count of all items if the client has good inventory controls and utilizes appropriate sampling techniques that are properly applied.28

Statistical Sampling for Nonauditing Uses

The independent auditor should be familiar with statistical techniques to aid him in his auditing procedures and to be able to understand them if he encounters them in his clients' operations. The auditor may also observe situations in which statistical techniques might reduce clerical costs or improve internal control for his client.

In this case, the auditor may very properly suggest to his client that he should use them. This lets the client know that the auditor is concerned about helping the client above the call of duty. This "extra service" will be beneficial to the auditor in having good client relationships and developing a successful practice.
BIBLIOGRAPHY


