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The Evolution and Application of Pi

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THE EVOLUTION AND APPLICATION OF PI

There is no part of the arithmetic that deals with approximations that is more interesting than that which seeks to find the ratio of the circumference of a circle to its diameter. This ratio has been studied from both a practical and a theoretical standpoint. Under the name "quadrature of the circle" it occupied mathematicians for many thousands of years, beginning with the Bible and extending to the twentieth century.

Many ancient people knew in a general way of this ratio but the first recorded use is found in the Old Testament with the description of Solomon's Temple in First Kings 7:21. King Hiram of Tyre made for the Temple a circular basin called "a molten sea." It was "ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it round about." From this π was ascertained to be equal to three.[1] The second mention of π, perhaps the oldest, (used here as the symbol for the ratio of the circumference of a circle to its diameter, but not established as a definite symbol until 1706 by William Jones)[2] is dated 3000 B.C. in the description of the pyramids of Egypt. The sides and heights of the pyramids of Cheops and Snefera at Gizeh are in the ratio 11:7, which

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[1] "Circle," World Book Encyclopedia (1959), 3, 2423.

[2] Cajori, Florian, A History of Elementary Mathematics, New York: The Macmillan Company, 1942, 324 pp.

the ratio of half the perimeter to the height 3 1/7.[3]

The next attempts to compute the value of π were often cast in the allied form of squaring the circle. The Rhind Papyrus of about 1700 B.C. provided a perfect example in A'h Mose's injunction to "cut one-nineth off the diameter and construct a square on it; its area will be equal to that of the circle." In other words, π is there declared to be $(4/3)^4$ or 3.1605, a most respectable value--one that is way-ahead, not only of Solomon's unwisdomly three, but more impressively of the approximation used by competent Roman engineers some sixty generations later.[4]

Next comes a long roll of Greek mathematicians who attacked the problem of π. Whether the researches of the the members of the Ionian School, the Pythagorians, Anazagoras, Hippias, Antipha, and Bryso led to numerical approximations for the value of π is doubtful, and their investigations are very unclear. The quadrature of certain lunes by hippocrates of Chios is ingenious and correct, but a value of π cannot be thence deduced; and it seems likely that the later members of the Athenian School concentrated their efforts on other questions. It is probable that Euclid, the illustrious founder of the Alexandrian School, was aware that π was greater than three and less than four, but he did not state the result explicitely.[5]

The first scientific attempt to compute π seems to be that of

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[3] Schepler, Herman C., Mathematics Magazine, 23 (January-February, 1950).

[4] Gridgeman, N. T., "Circumetrics," Scientific Monthly, 77 (July, 1953), 31-5.

[5] Ball, W. W. Rouse, Mathematical Recreations and Essays, New York: The Macmillan Company, 1947, 418 pp.

Archimedes. In his book Measurement of the Circle, Archimedes presents three propositions suggesting the value of π.[6] He proves first that the area of a circle is equal to that of a right triangle having the length of the circumference for its base, and the radius for its altitude. In this he assumes that there exists a straight line equal in length to the circumference--an assumption objected to by some ancient critics, on the ground that it is not evident that a straight line can equal a curved one. The finding of such a line was the next problem. He first finds anupper limit to the ratio of the circumference to the diameter, or π. To do this, he starts with an equilateral triangle of which the base is a tangent and the vertex is the centre of the circle. By successively bisecting the angle at the centre, by comparing ratios, and by taking the irrational square root always a little too small, he finally arrived at the conclusion that $\pi = 3\ 1/7$. Next he finds a lower limit by inscribing in the circle regular polygons of 6, 12, 24, 48, 96 sides, finding for each successive polygon its perimeter, which is always less than the circumference. Thus he finally concludes that "the circumference of a circle exceeds three times its diameter by a part which is less than 1/7 but more than 10/71 of the diameter."[7]

Hero of Alexandria gave the value three, but he quoted the result 22/7: possibly the former number was intended only for rough approximation.[8]

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[6] Ball, W. W. Rouse, A Short Account of the History of Mathematics, New York: The Macmillan Company, 1924, 522 pp.

[7] Cajeri, Florian, A History of Mathematics, New York: The Macmillan Company, 1919, 514 pp.

[8] Ball, op. cit. (5).

Ptolemy (c. 150) seems to have taken the Archimedean limits and to have expressed them in sexagesimals obtaining substantially 3 1/7 = $3^{\circ}8'34.28''$ and 3 10/71 = $3^{\circ}8'27.04''$. He then improved upon the mean between these results by taking $3^{\circ}8'30''$ as the approximate value of π, although a still closer approximation is $3^{\circ}8'29,73355''$. Since $3^{\circ}8'30'' = 3.1416$, his result was very satisfactory.[9] This represents an error of roughly 25 parts per million, equivalent to about two-thirds of a mill in the circumference of the earth, and it prompts the question of why has its ultra-refinement been so zealously pursued through the ages that followed? An adequate answer could not be attempted here; it is too complex; but it may be noted that the question is bound up with the scholastic interregnum of the Dark Ages, with the weaknesses of communicationa, with the belief that π was a sort of philosopher's stone of mathematics, and even with the spirit that sends men to the toil of Mount Everest. The interregnum meant a virtual cessation of π work in the West for roughly one thousand years; and the communication weaknesses linked up with the formidable linguistic difference between cultures that obscured from everyone, until comparatively recently, the pi-istic contributions of the Far East. Additionally, the all too common Asiatic inclination to fuse science and Holy Writ, and the twisting of expression to meet the demands of versification--especially in India--combined to fuzz the picture. Today however, there is bulky literature on the topic from which several will be mentioned. For example, in A.D. 479, Tau Tung Chih put forward the fraction 355/113, which is correct to six decimal places (which was

[9] Smith, D. E., History of Mathematics, New York: Dover Publications, 1953, 735 pp.

exported to Japan, and was to turn up in sixteenth century Europe in the writings of Otho and Metuis). A hundred years later Arya Bhata the Elder's work with polygons led to the recipe "add 4 to 100, multiply by 8, add 62,000, and the result is the circumference of a circle of diameter 20,000," which implies $\pi = 3.1416$.[10] Brahmagupta (c.628) criticized Arya Bhata for taking the circumference as 3393 for both diameters 1080 and 1050, which would make π either 3 17/120 or 3 81/350, that is 3.1416 or 3.2314. A certain astronomer, Pulisa, to whom Brahmagupta referred gave 3 177/1250, which is 3.18+, and Bhaskara (c.1150) used 3927/1250 for the "near" value and 22/7 in finding the "gross circumference adapted to practice," the former being the same as the value 3 177/1250 of Pulisa.

The Chinese found various values of π , but the methods employed by the early calculators are unknown. The value 3 was used probably as early as the twelfth century B.C. and is given in the <u>Chou-pei and the Nine Sections</u>. Ch'ang Hong used $\sqrt{10}$, and Wang Fan used 142/45, which is equivalent to 3.1555... Lui Hui gave the first intimation of the method used by the Chinese in finding the value. He began with a regular inscribed hexagon, doubles the number of sides repeatedly, and asserts that "if proceeding until the process of doubling can no longer continue, the perimeter ultimately comes to coincide with the circle."

The Japanese did no noteworthy work in the field until the seventeenth century. They then developed a kind of native calculus and also made use of European methods which gave them fair approximations to the required ratio.[11]

During the following years an approximate value of π was obtained

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[10] Gridgeman, op. cit.

[11] Smith, op. cit.

experimentally by the theory of probability. It was during this period that Comte de Buffon devised his famous needle problem by which he determined π . Suppose a number of parallel lines, distance (a) apart, are ruled on a horizontal plane, and suppose a homogeneous uniform rod of length l ·a is dropped at random onto the plane. Buffon showed that the probability that the rod will fall across one of the lines in the plane is given by $p=2l/$:a. By actually performing this experiment a given large number of times and noting the number of successful cases, thus obtaining an empirical value for p, the above formula may be used to compute π . The best result obtained in this way was given by the Italian, Lazzerini, in 1901. From only 3408 tosses of the rod he found π correct to 6 decimal places. His result is so much better than those obtained by other experimenters that it is sometimes regarded with suspicion.[12]

Other similar methods of approximating the value of π were developed. For instance, it is known that if two numbers are written down at random, the probability that they will be prime to each other is $6/\pi^2$. Thus, in one case where each of 50 students wrote down five pairs of numbers at random, 154 of the pairs were found to consist of numbers prime to each other. This gives $6/\pi^2 = 154/250$, from which $\pi \neq 3.12$.[13]

Prevading much early speculation and measurement is the belief that π is a rational quantity, that it can be exactly rendered as a vulgar fraction. The Greeks were fully aware of the concept of irrational quantities, test Pythagora's celebrated proof of the irrationality of $\sqrt{2}$,

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[12] Eves, Howard, An Introduction to the History of Mathematics, New York:. Holt, Rinehart, and Winston.

[13] Ball, op. cit. (5).

but it was not until 1762 that π was put into that category by Lambert (and thirty-three years later Legendre made the important extension to π^2), and even those who felt in their bones that a simple fraction was unrealizable could not be blamed for putting their money on some not too cumbrous expression that would be π. Such as, for instance, Kochansky's $\sqrt{4 + (3 - \sqrt{1/3})^2}$ yielding 3.14153, and Specht's $1/10^7 + \sqrt{146} \times 13/50$ which is correct to fourteen decimals.

An important transition in pi-valuation took place during the Renaissance, namely, the replacement of polygonning by the setting up of infinite series that could be calculated to whatever degree of accuracy a man cared to give his time. Probably the first formulation was $2/\pi = \sqrt{2}/2 \cdot \sqrt{2+\sqrt{2}}/2 \cdot \sqrt{2+\sqrt{2+\sqrt{2}}}/2 \ldots$ Nevertheless, polygon devotees throve for long enough afterwards. Ludolph van Ceulen, for example, began about the time of Vieta's discovery to lay aside all other activities to make piiferous polygonning a full-time occupation. By the turn of the century he had established 20 decimal places. When he died in 1610 the number was thirty-five places, which value of π was inscribed on his tombstone at Leyden. In 1630 Grienberger published the thirty-nine places of π he had sweated out using a simpler process than van Ceulen's. His was the last effort of note. The seventeenth century witnessed the development of convergent infinite series and fractions. Gregory and Leibniz evolved the series $\pi/4 = 1/(2+3^2) / (2+5^2) / (2+7^2)/\ldots$ that was later to become the basis of modern rapidly converging series.[14]

The first analytical expression discovered in the seventeenth century is the infinite product $\pi/2 = 2/1 \cdot 2/3 \cdot 4/3 \cdot 4/5 \cdot 6/5 \ldots$

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[14]Gridgeman, op. cit.

which was published by John Wallis in 1655.

Lord Brouncker, the first president of the Royal Society, about 1658 found the infinite continued fraction $\pi/4 = 1/1+ \cdot 1^2/2+ \cdot 3^2/2+ \ldots$ which was shown subsequently by Euler to be equivalent to the alternating series $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9+ \ldots$ known to G. W. Leibniz in 1674.

The great majority of calculations of π to many decimal places have been based upon the power series $\arctan x = x - x^3/3 + x^5/5 - \ldots$, $-1 \leqq x \leqq 1$, which was discovered in 1671 by James Gregory. He failed, however, to note explicitely the special case corresponding to $x = 1$, which is ascribed to Leibniz.

Sir Isaac Newton in 1676 discovered the power series $\arcsin x = x + 1/2 \cdot x^3/3 + 1/2 \cdot 3/4 \cdot x^5/5+ \ldots\ -1 \leqq x \leqq 1$, which has been used by a few computers of π .

In 1755 Leonhard Euler obtained the following series: $\arctan x = x/1+x^2 \left\{1+2/3\ (x^2/1+x^2) + 2.4/3.5\ (x^2/1+x^2)^2 + \ldots\right\}$.

It was by means of Gregory's series, taking $x=1/\ 3$, that Abraham Sharp, at the suggestion of the English astronomer Edmund Halley, computed π to 72 decimal places in 1699, thereby nearly doubling the greatest accuracy attained by earlier computers, who had used geometrical methods. Sharp's calculation was extended by Fautet de Lagny in 1719 to 127 decimals. (the 113th place has a unit error).

Newton set $x=1/2$ in his series, and thereby computed π to 14 places. A Japanese computer, Matsunaga Ryohitsu, used the same procedure to evaluate π correct to 49 decimal places in 1739. About 1800 a Chinese, Chu Hung, calculated π to 40 places (25 correct) by this series.

Most computer's of π in modern times have used Gregory's series in conjunction with certain arctan relations. Only nine of these relations have been employed to any extent in such computations. We shall now consider these formulas, arranged according to the increasing precision of the approximation computed by their use.

I. $\pi/4 = 5 \arctan 1/7 + 2 \arctan 3/79$

Euler in 1755 used this relation in conjunction with his series for arctan x to compute π correct to 20 decimal places in one hour. Baron Georg von Vega in 1794 employed Gregory's series and the preceding relation to evaluate π to 140 decimal places, of which the first 136 were correct. This precision was exceeded by that attained by an unknown calculator whose manuscript, containing an approximation correct to 152 places, was seen in the Radcliffe Library at Oxford toward the close of the 18th century.

II. $\pi/4 = \arctan 1/5 - \arctan 1/70 + \arctan 1/99$

Euler published this relation in 1764. It was used by William Rutherford in 1841 to compute π to 208 places (152 correct).

III. $\pi/4 = \arctan 1/2 + \arctan 1/5 + \arctan 1/8$

This formula was supplied the calculating prodigy Zacharias Dahse by L. K. Schulz von Strassnitzky of Vienna. Within a period of 2 months in 1844, Dahse thereby evaluated π correct to 200 places.

IV. $\pi/4 = \arctan 1/2 + \arctan 1/3$

First published by Charles Hutton in 1776, this relation was used by W. Lehmann of Potsdam to compute π to 261 decimals in 1853. Tseng Chi-hung in 1877 used the same formula to evaluate π to 100 decimals in a little more than a month.

V. $\pi/4 = 2 \arctan 1/4 + \arctan 1/7$

The relation was also published by Hutton in 1776, and independently by Euler in 1779. Vega used it in 1789 to compute 143 decimals (126 correct). In order to remove the uncertainity caused by the disciepant approximation of Rutherford and Dahse, Thomas Clausen extended the calculation to 248 correct decimals in 1847, and Lehman reached 261 decimals in 1853 by this formula, confirming his independ calculation of π to the same extent by relation IV. Edgar Frisby in Washington, D.C., used relation V in conjunction with Euler's series to compute π to 30 places in 1872.[15]

In 1882 Lindemann, proved π to be transcendental, thus destroying forever the last slim hope of those who would square the circle. A number which is not algebraic is transcendal, that is one which satisfies no algebraic equation with complex number coefficients, which is exactly what mathematicians for centuries had been proving about pi.[16]

In 1897, at which time the people fo the United States were already displaying their abilities in the various sciences, a bill was introduced in the legislature of the state of Indiana to set the value of π equal to a simple, easily managed, fraction. The bill was not written by a madman, or a lunatic, or an organization of foreign spies. The bill was written by a country medical doctor who fancied himself somewhat of a mathematician. So the bill was written, suggesting several different simple (but incorrect) relationships between the area of a circle, the circumference

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[15] Wrench, J. W., "The Evolution of Extended Decimal Approximations to Pi," School Science and Math, 60 (May, 1960), 348-350.

[16] Bell, Mathematics Queen and Servant of Science, New York: Mc-Graw-Hill Book Company, 1951.

of that circle, the diagonal of a square, etc.[17]

House Bill No. 246

A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the state of Indiana free of cost by paying any royalties whatever on the same, provided it is accepted and adopted by the official action of the legislature of 1897.

Section 1: Be it enacted by the General Assembly of the State of Indiana, that it has been found that a circular area is to the square on a line equal to the quadrant of the circumference as the area of an equilateral triangle is to the square on one side. The diameter employed as the linear unit according to the present rule in computing the circle's area is entirely wrong, as it represents the circle's area one and one-fifths times the area of a square whose perimeter is equal to the circumference of the circle. This is because one-fifth of the diameter fails to be represented four times in the circle's circumference. For example, if we multiply the perimeter of a square by one-fourth of any line one-fifth greater than one side, we can in like manner make the square's area to appear one-fifth greater than the fact, as is done by taking the diameter for the linear unit instead of the quadrant of the circle's circumference.

Section 2: It is impossible to compute the area of a circle on the diameter as the linear unit without trespassing upon the area outside of the circle to the extent of including one-fifth more area than is contained within the circle's circumference, because the square on the diameter produces the side of a square which equals nine when the arc of ninety degrees equals eight. By taking the quadrant of the circle's circumference for the linear unit, we fulfill the requirement of both quadrature and rectification of the circle's circumference. Furthermore it has revealed the ratio of the chord and arc of ninety degrees, which is as seven to eight and

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[17]Greenblatt, M. H., "Pi-three Versus Pi-four," Math Teacher, 62 (March, 1969), 223-225.

> also the ratio of the diagonal and one side of a square which is as ten to seven, disclosing the fourth important fact, that the ratio of the diameter and circumference is as four-fifths to four; and because of these facts and the further fact that the rule in present use fails to work both ways mathematically, it should be discareded as wholly wanting and misleading in its practical application...[18]

House Bill No. 246 passed the House, but because of newspaper ridicule, was shelved by the Senate.

VI. $\pi/4 = 3 \arctan 1/4 + \arctan 1/20 + \arctan 1/1985$

This formula was published by S. L. Lanwy in 1893, by Carl Stormer in 1896, and was rediscovered by R. W. Morris in 1944. By means of this formula D. F. Ferguson, then of the Royal Naval College, performed a longhand calculation of π to 530 decimal places between May 2944 and May 1945. At that time he discovered a discrepancy between his approximation and the final result of William Shankd beginning with the 528th place. The first notice of an error in Shank's well-known approximation appeared in a note published by Ferguson in March 2946. He continued his calculation of π and in July 1946 published a correction to Shank's value through the 620th decimal place. Subsequently, Ferguson used a desk calculator to reach 710 decimals by January 1947, and finally 808 decimals by September 1947.

VII. $\pi/4 = 8 \arctan 1/10 - \arctan 1/239 - 4 \arctan 1/515$

S. Klingenstierna discovered this relation in 1730; it was rediscovered more than a century later by Schellbach. It was used by C. C. Camp in 1926 to evaluate $\pi/4$ to 56 palces. D. H. Lehmer recommended it

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[18] Read, Cecil B., "Historical Oddities Relating to the Number Pi," School Science and Math, 60 (May, 1960), 348-50.

in conjunction with the next formula for the calculation of π to many figures. G. E. Felton on March 31, 1957 completed a calculation of π to 10021 places on a Pegasus computer at the Ferranti Computer Centre in London. This required 33 hours of computer time. The result was published to 10000 places. A check calculation using formula VIII revealed that, because of a machine error, this result was incorrect after 7480 decimal places.

Gauss investigated the derivation of arctan relations and reduced it to a problem in Diophantine analysis. Relation VIII is one of several formulas he developed. J. P. Balkantine substantiated Lehmer's claim that this formula is especially effective for extensive calculation, by discussing its use in conjunction with Euler's series for the arctan.

VIII. $\pi/4 = 12 \arctan 1/18 + 8 \arctan 1/57 - 5 \arctan 1/239$

Felton carried out a second calculation to 10021 places, and by May 1, 1958 had removed all discrepancies from his results, so that the approximation computed from formulas VII and VIII agreen to within three units in the 10021st decimal place. The corrected result remains unpublished.

IX. $\pi/4 = 4 \arctan 1/5 - \arctan 1/239$

This is the most celebrated of all the relations of this king. John Machin, its discoverer, computed π correct to 100 decimals by means of it in conjunction with Gregory's series and the result appeared in 1706. Clausen in 1847 used this relation in addition to Hutton's formula V to compute π correct to 248 decimal places, as has already been noted.

Rutherford resumed his calculation of π in 1852, using Machin's formula this time, as did his former pupil William Shanks. Shank's first published approximation to π contained 530 decimal places, and was incorporated in Rutherford's note, published in 1853, which set forth his approximation to 441 decimals. Later that year Shanks published his book containing an approximation to 530 places. It is now known that Shank's value was incorrectly calculated beyond 527 places. The accuracy of that value was further vitiated by a blunder committed by Shanks in correcting his copy prior to publication, with the result that similar erros appeared in decimal places 460-462 and 513-515. These erros persist in Shanks first paper of 1873 containing the extension to 707 decimals of his earlier approximation. His second paper of that year, which contained his final approximation to π, gives corrections of these errors; however, there appears an inadvertent typographical error in the 326 place of his final value. In retrospect, we now realize that his first value published in 1853 was the most accurate he ever published.

The accuracy of Shank's approximation to at least 500 decimals was confirmed by the independent calculations of Professor Richter of Elbing, Gr., who in 1853-1854 computed successive approximations to 330, 400 and 500 places. Richter's communications do not reveal the formula that he used.

Machin's formula was used by H. S. Uhler in an unpublished computation correct to 282 places, which was completed in August 1900.

F. J. Duarte computed π correct to 200 places by this method in 1902. The result was published six years later.

As a by-product of his calculation of the natural logarithms of small primes, Uhler in 1940 noted confirmation to 333 places of Shank's approximation.

In December 1945, Professor R. C. Archibald suggested that the writer undertake the computation of π by Machin's formula in order to provide an independent check of the accuracy of Ferguson's calculations. With the collaboration of Levi B. Smith, who evaluated arctan 1/239 to 820 places, the writer computed π to 818 places by February 1947, using a desk calculator. The result was published to 808 places in April, 1947, and was verified to 710 places by Ferguson in a note published concurrently. The limit of 808 places was chosen to probide precision comparable to that obtained by P. Pedusen in his approximation of π.

Collation of this 808 place approximation with the results obtained by Ferguson later that year revealed several erroneous figures beyond the 723 place in the writer's approximation to arctan 1/5. These errors vitiated the corresponding figures in the approximation of π. Corrections of these errors and extensions of Ferguson's results appeared in a joint paper by Ferguson and the writer in January 1948, which concluded with an 808 place approximation of π of guaranted accuracy.

Subsequently, Smith and the writer resumed their calculations and by June 1949 had obtained an approximation to about 1120 places. Before final checking of this extension could be completed, the ENIAC at the Ballistic Research Laboratories, Aberdeen Proving Ground, was employed by George W. Rutiviesner and his associates in September 1949 to evaluate to about 2037 places in a total of 70 hours. Machin's formula was also used in this computation.

In November 1954, Smith and the writer extended their calculations to 1150 places and in January 1956 reverted to this work once more to attain their final result, which was terminated at 1160 places, of which the first 1157 agree with ENIAC.

A calculation of π was performed in duplicated on the NORC (Naval Ordiance Research Calculator) in November 1954 and in January 1955 as a demonstration problem, prior to the delivery of that computed to the U. S. Naval Proving Grounds. Again, Machin's formula was selected, and the calculation was completed to 3093 places in 13 minutes running time.

In January 1958, Francois Genuys programmed and carried out the evaluation of π correct to 10000 places on an IBM 704 Electronic Data Processing System at the Paris Date Processing Center. Machin's formula in conjunction with Gregory's series was used. Only 40 seconds were required to attain the 707 decimal place precision reached by Shanks and one hour and forty minutes was required to reach the 10000 places of the final result.

On July 30, 1959, the program of Genuys was used on an IBM 704 system at the Commissariat a l'Energie Atomique in Paris to compute pi to 16167 decimal places. This latest approximation in unpublished at present.[19]

If the problem of squaring the circle had not focused the interest of mathematicians on the number π , they would have been obliged to find an approximate value for this number because of its importance in other connections. Pi is no longer an unknown that is to be accepted but a proven number that is to be calculated and used. The motivation of modern and pre-modern calculators of pi will be passed to generations to come with the understanding that they also will prove, disprove, challenge, and accept the many theories and values of pi.

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[19] Wrench, op. cit.

CONTRIBUTORS TO THE VALUE OF PI

Date	Contributor	Nation	Value
3000 B.C.	Pyramids	Egypt	3 1/7
2000 B.C.	Rhind Papyrus	Egypt	16/9
950 B.C.	Bible	Hebrew	3
240 B.C.	Archimedes	Greek	between 3 10/71 & 3 1/7
100 B.C.	Heron	Alexandria	3 1/7 or 3
20 B.C.	Vitruvius	Italy	3 1/8
25 A.D.	Luis Hsiao	China	3.16
97 A.C.	Sextus Julius Frontinus	China	3 1/7
125 A.D.	Ch'ang Hong	China	$\sqrt{10}$
150 A.D.	Ptolemy	Alexandria	3 17/120
250 A.D.	Wang Fan	China	142/45
263 A.D.	Lui Hui	China	157/50
450	Wo	China	3.1432
480	Tsu Ch'ung-chih	China	between 3.1415926 & 3.1415927
500 A.	Arya-Bhata	India	between 3.177/1250 & 62832/20000
505	Varahamihira Pancha Siddhantika	India	$\sqrt{10}$
530	Baudhayana	India	49/16
628	Bsahmagupta	India	22/7
825	al-Khowarizmi	Arabia	22/7, $\sqrt{10}$, 62832/20000
850	Mahavira	India	$\sqrt{10}$
1150	Bhaskara	India	3927/1250, 22/7, $\sqrt{10}$, 754/240
1220	Leonardo Pisano	Italy	1440/458
1260	Johannes Campanero	Italy	22/7
1430	Al Kashi	Persia	3.14159265358987 32

1460	Georg von Peubach	Austria	62832/20000
1464	Nicolaus de Cusa	Germany	$3/4\ (\sqrt{3}+\sqrt{6})$
1464	Regiomontanus	Germany	3.14343
1525	Stifel	Germany	3 1/8
1573	Valentin Otto	Germany	355/113
1579	Vieta	France	9 places
1580	Tycho Brahe	Denmark	$88/\sqrt{785}$
1585	Simon van der Eycke	Netherlands	3.1416055
1585	Andriaen Anthonisz	France	355/113
1593	Andriaen	Netherlands	17 places
1596	Ludolph van Ceulen	Germany	20 places
1610	Ludolph van Ceulen	Germany	34 places
1630	Grienberger	Italy	39 places
1650	John Wallis	England	3.14
1654	Christian Huygens	Netherlands	9 places
1666	Thomas Habbis	England	3 1/5
1666	Sato-Seiko	Japan	3.14
1668	Gregory	Scotland	3.14
1673	Leibnitz	Germany	3.14
1685	Father Kochansky	Poland	3.1415333
1690	Takibe	Japan	41 places
1699	Sharp	England	72 places
1706	Machin	England	100 places
1719	Lagny	France	127 places
1720	Matsunaga Ryohitsu	Japan	50 places
1753	M de Causans	France	4
1760	Count de Buffon	Germany	3.1415929
1776	Hesse	Germany	3 14/99
1789	George Vega	Austria	143 places
1794	George Vega	Austria	140 places
1800	Chu Hung	China	40 places
1825	Malacarne	Italy	$\pi < 3$
1828	Specht	Germany	3.141591953
1833	William Baddeley	England	3.202216

1836	La Comme	France	3 1/8
1837	J. F. Callet	France	154 places
1841	Rutherford	England	208 places
1844	Dase	Germany	205 places
1847	Thomas Clausen	Germany	250 places
1849	Jakob de Gelder	Germany	3.14159292035
1851	John Parker	U.S.A.	20612/6561
1853	Rutherford	England	440 places
1853	Shanks	England	607 places
1853	Richter	Germany	333 places
1860	James Smith	England	3 1/8
1862	Lawrence Benson	U.S.A.	3.141592
1863	S. M. Drach	England	3.14159265000...
1868	Cyrus Pitt Grosvenor	U.S.A.	3.142135
1873	Shanks	England	707
1876	Alick Carrick	England	3 1/7
1877	Tsung Chi-hung	China	100 places
1879	Pliny E. Chase	U.S.A.	3.14158499
1892	N. Y. Tribune	U.S.A.	3.2
1902	E. W. Hobson	England	3.14164079
1913	F. T. Ovarte	French	200 places
1913	Srinivasa Ramaniyan	India	3.14159265258 26
1914	T. M. P. Hughes	England	3.14159292035
1928	Gottfried Lonzer	U.S.A.	3.1378
1933	Helen A. Merrill	U.S.A.	3.141591953
1934	Heisel	U.S.A.	3 13/81
1944	D. F. Ferguson	England	530 places
1945	Levi B. Smith, Dr. John Wrench, Jr.	U.S.A.	818 places
1949	ENIAC	U.S.A.	2037 places
1954	Levi B. Smith, Dr. John Wrench, Jr.	U.S.A.	1150 places
1955	NORC	U.S.A.	3093 places
1958	Francois Genuys	French	10,000 places
1959	Commissariat at' Energie Atomique	French	16,167 places
1961	Dr. Daniel Shanks, Dr. Joyn Wrench, Jr.	U.S.A	100,265 places

SHANKS 707 DECIMAL PLACES OF PI

Computed in 1873

3.14159	26535	89793	23846	26433	83279	50288	41971	69399
37510	58209	74944	59230	78164	06286	20899	86280	34825
34211	70679	82148	08651	32823	06647	09384	46095	50582
23172	53594	08128	48111	74502	84102	70193	85211	05559
64462	29489	54930	38196	44288	10975	66593	34461	28475
64823	37867	83165	27120	19091	45648	56692	34603	48610
45432	66482	13393	60726	02491	41273	72458	70066	06315
58817	48815	20920	96282	82540	91715	336436	78925	90360
01133	05305	48820	46652	13841	46951	94151	16094	33057
27036	57595	91953	09218	61173	81932	61179	31051	18548
07446	23798	34749	56735	18857	52724	89122	79381	83011
94912	98336	73362	44193	66430	86021	39501	60924	48077
23094	36285	53096	62027	55693	97986	95022	24749	96206
07497	03041	23668	86199	51100	89202	38377	02131	41694
11902	98858	25446	81639	79990	46597	00081	70029	63123
77381	34208	41307	91451	18398	05709	85		

DESIGNED PROGRAM FOR COMPUTER COMPUTATION OF

$\pi = 1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 - 1/17 - 1/19 + \ldots$

```
      Program Series (INPUT, OUTPUT)
      DIMENSION FSIGN (17), SPRIME (5500), IPRIME (5500)
      DOUBLE SUM, A, B
      READ I, IA
1     FORMAT (I5)
      SPRIME (1) = +1.0
      SPRIME (2) = +1.0
      IPRIME (1) = 2
      IMPRIE (2) = 3
      B = 3.000
      SUM = 1.8333333333333333
      KSCNT = 3
      M = 2
      DO 200 K = 4, IA
      B = B + 1.000
      A = 1/B
      Y = 1.000 * K
      C = SQRT (Y) + 1.000
      IE = INT (C)
      L = 1
2     L - L + 1
      IF (IE.GT.L) 3, 5
3     IC = K/L
      IB = IC * L
4     IF (K.EQ.IB) 75, 2
5     IH = K + 1.
      IPDGN = 1.
6     IG = IH/4
```

```
      ID = 4 * IG
 7    IF (IH.EQ.ID) 89
 8    M = M + 1
      SPRIME (M) = +1.0
      IPRIME (M) = K
      ASIGN = SPRIME (M)
      GO TO 100
 9    M = M + 1
      SPRIME (M) = +1.0
      IPRIME (M) = K
      ASIGN = SPRIME (M)
      GO TO 100
75    J = 1
      IPDGN = 0
11    DO 12 I = 1, 17
      FSIGN (I) = +1.0
12    CONTINUE
      I = 0
      IR = K
13    N = IPRIME (J)
      J = J + 1
14    IQ = IR/N
      IP = IQ * N
15    IF (IR.EQ.IP) 16, 13
16    I = I + 1
      IR = IQ
      FSIGN (I) = SPRIME (J - 1)
      IF (IR.EQ.1) 17, 14
18    TERM = +1.0
      DO 18 I = 1, 17
      TERM = TERM * FSIGN (I)
18    CONTINUE
      ASIGN = TERM
100   SUM = SUM + ASIGN * A
      IF (ASIGN.EQ+1.0) 50, 101
```

```
50      KSCNT = KSCNT + 1.
101     IT = K/50
        IV = 50 * IT
        IF (K.EQ.IV) 175, 200
175     PRINT 19, K, PDGN, M, ASIFN, A, SUM
19      FORMAT (//X, I5,6X,I1,6X,I4,6X,F6.1,10X,D40.28,10X,D40.20)
        RNUM = 1.000 *KSCNT
        RDNM = 1.000 * K
        RATIO = RNUM/RDNM
176     PRINT 177, KSCNT, RATIO
177     FORMAT (1X, 10HPOSITIVES =,I5,6X,6HRATIO=,F24.14)
200     CONTINUE
        END
        STOP
```

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