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The Mathematical and Historical Significance of the Four-Color Theorem

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The Mathematical and historical significance of the Four Color Theorem

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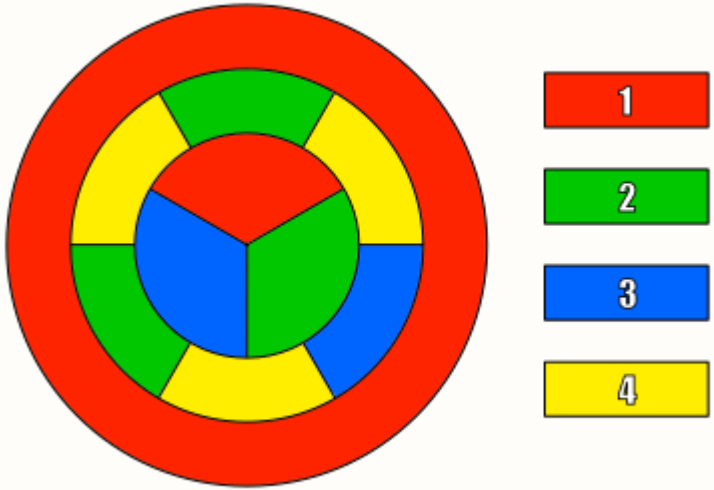
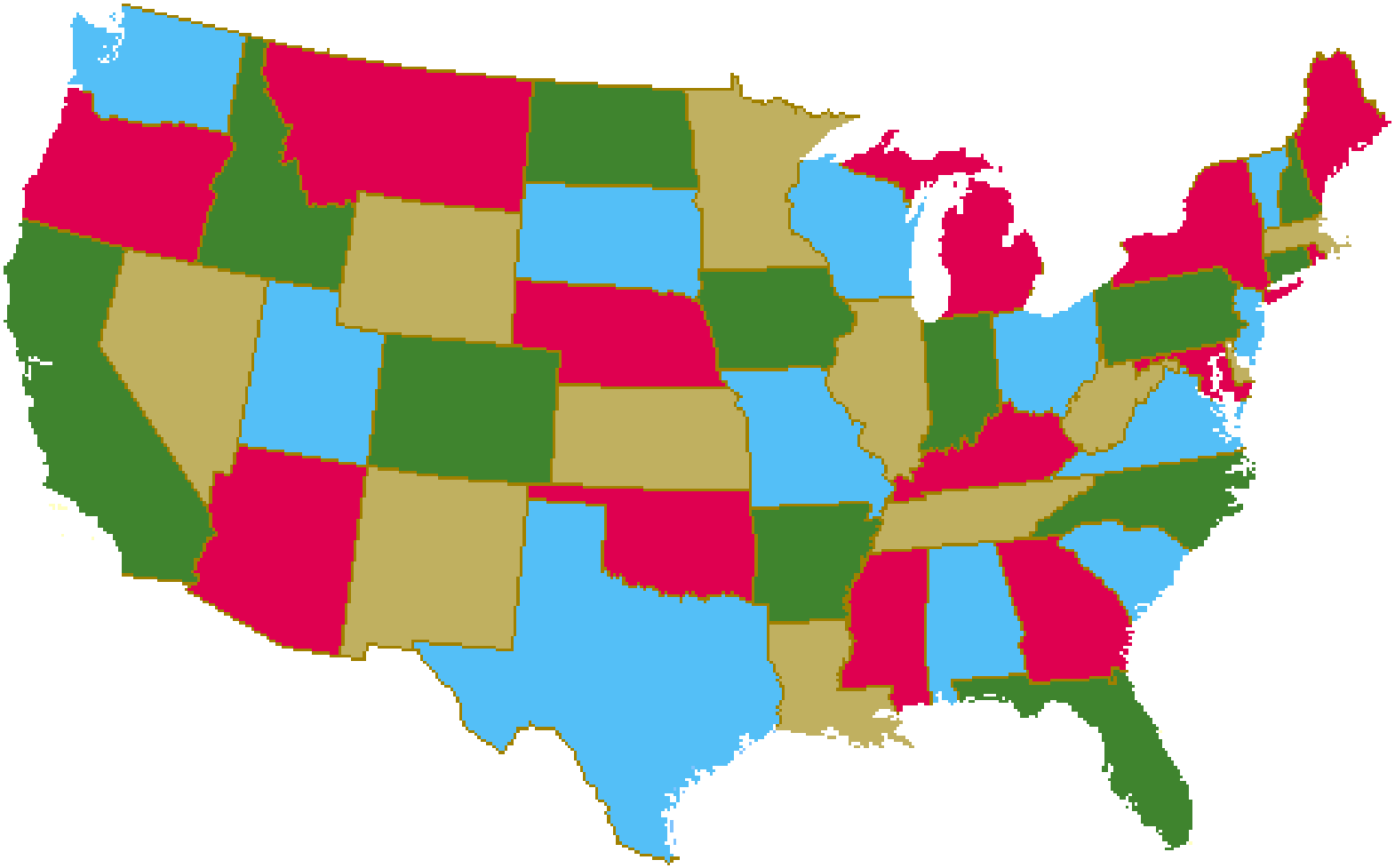
Introduction

Apply problem -solving strategies in various contexts

Introduction

To explain the Four Color Theorem, I have included a picture to our right. The theorem says that given any **2 dimensional plane** , you can color the regions with **4 colors or less** such that no two adjacent regions are colored the same.





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Background + Proof

Historical Background

We begin our journey of the Four Color Theorem all the way back in **1852**, where **Francis Guthrie** originally stated the conjecture for the problem that was titled **“Guthrie’s Problem”** at the time. Then, after that work that Guthrie conjectured, another mathematician by the name of **Arthur Cayley** wrote the first paper on this topic, where he expressed the major difficulties that proving this problem could have. Then, just a short year later another mathematician by the name of **Alfred Bray Kempe** published a solution that was included in the **1879** issue of the *American Journal of Mathematics* and this solution to Guthrie’s original problem was accepted for a little bit, until **in 1890**, yet another mathematician by the name of **Pearcy John Heawood** discovered a fatal flaw in Kempe’s proof. This would ultimately leave the problem unsolved for around another 86 years

What is a Proof?

A mathematical proof, simply, is an argument that something **is true**. The situations may vary, but a mathematical proof uses **logic** to deduce something about a given context. For example, how would one proof that $\sqrt{2}$ is irrational?


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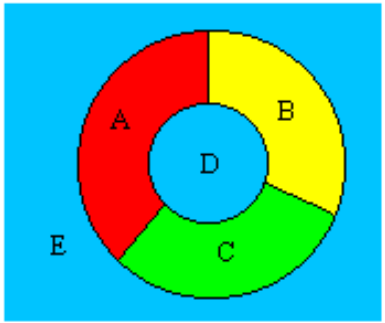
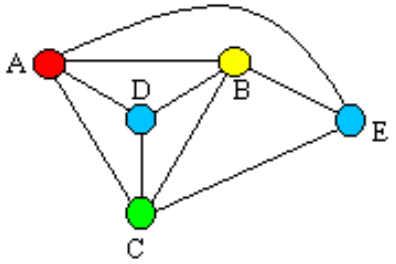
The Proof of the Four Color Theorem

But, in 1977, two mathematicians studying at the University of Illinois would come up with a solution. **Dr. Kenneth Appel** and **Dr. Wolfgang Haken** would ultimately publish a proof of the Four Color theorem, but how did they do it? They would use a branch of mathematics called **Graph Theory** .

Graph Theory

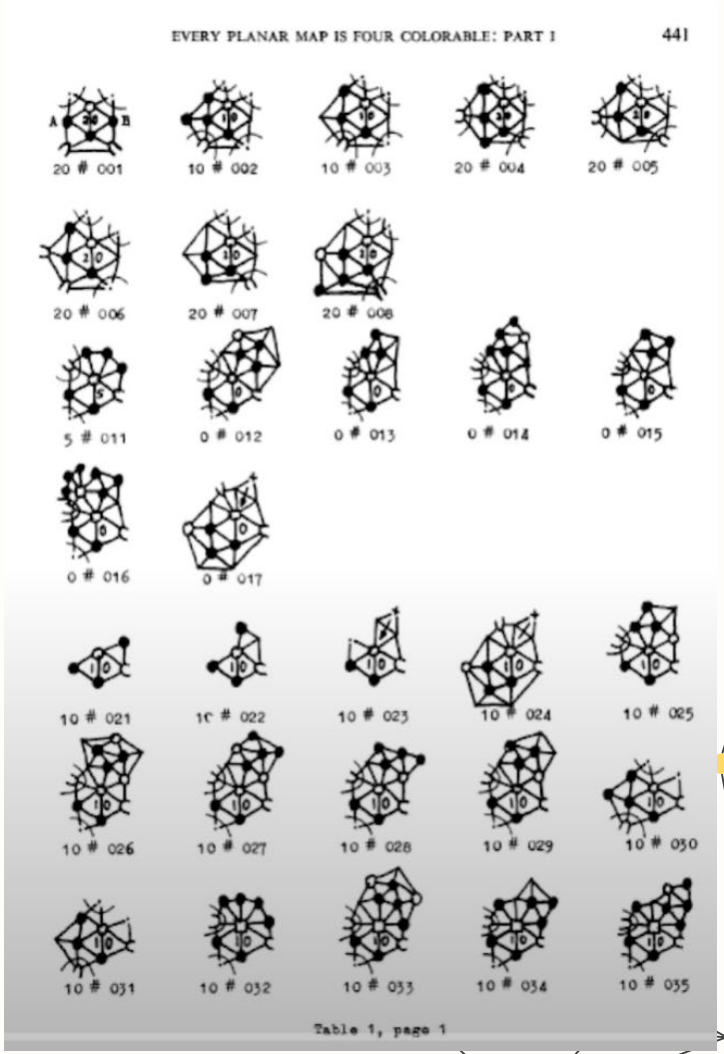
The whole idea of **Graph Theory** is to study the **vertices** and **edges** of a graph. Appel and Haken would use this branch of Mathematics to turn their picture into a graph. Shown to the right is an example of a 2 dimensional shape turned into a graph theory problem.





Their Proof Strategy

Appel and Haken would use a strategy derived by Kempe, which utilized **Kempe Chains**. These Kempe chains were used to describe similar combinations of graphs. Overall, they would describe all possible combinations of these vertices and edges to use a proof technique known as "**Proof by Exhaustion**."



Strategy Continued.

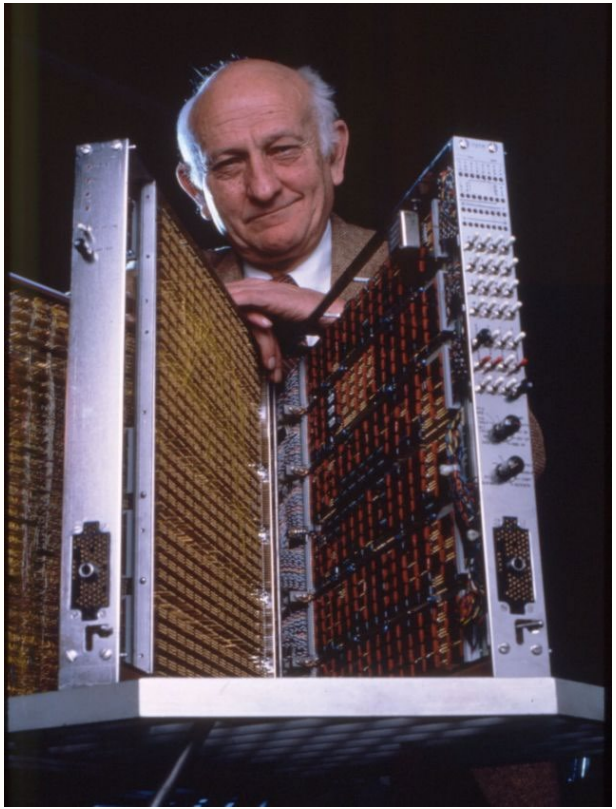
Appel and Haken would have to check **over 1900 different cases** in order to fully prove the Four Color Theorem. This felt like way too many cases to prove by hand, so they tried something never before done in Mathematics. They used a **computer** .

Computer Specifics

Appel and Haken utilized the T eletype Model ASR - 33 which only ran at a staggering 110 bits per second, which if you put that into perspective, would take that computer 105 days to download a 1 gigabyte file. So, because it would have taken Appel and Haken exponentially longer to compute all of these cases by hand, they utilized this computer in order to prove the Four Color Theorem.

How they proved the FCT

With the help of **John Cocke**, an IBM Researcher, who was on the team that developed the compiling system for their computer, they were able to **more efficiently** check the cases as well as publish a proof of the Four Color Theorem for the first time in history.



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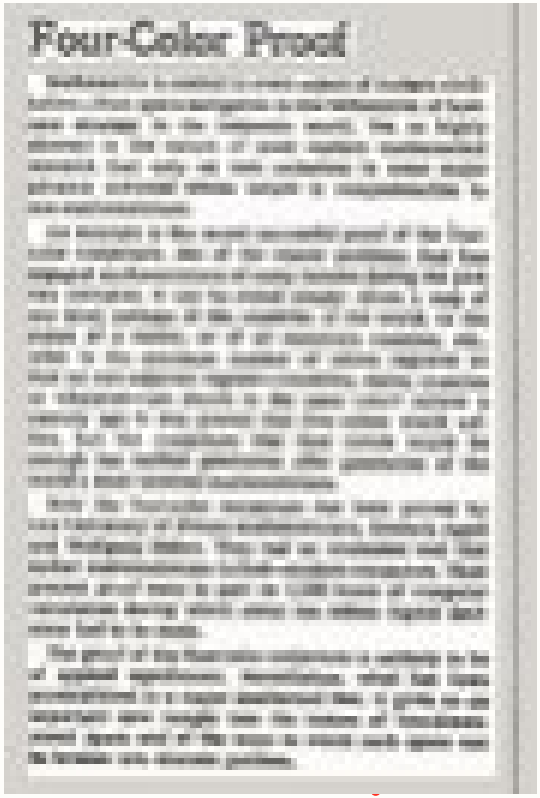
The Controversy

Chaos

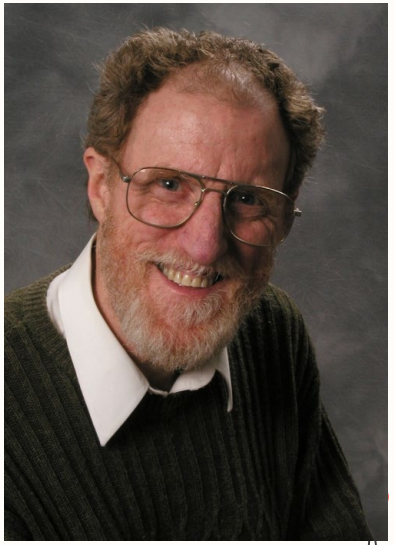
This would be where the chaos would erupt. Many older mathematicians at that time didn’t want to have computers in mathematics and believed that they should be separate, while some younger mathematicians were all for computer aided proofs. The mindset behind these older mathematicians was that since the discovery of mathematics, mathematicians would use pen and paper to write down their proofs and use their own brain to figure out these complex problems. How could we stoop so low as to use a computer’s brain to figure out a problem? If a human couldn’t check the validity of a proof in the first place, is the proof ever even true?

New York Times Criticism

One article I read even stated that the New York Times straight up **refused to publish any news** about this proof just because of the fact that "it was going to be wrong anyway." How could you trust a program that does all of these computations but can't explain the reasonings behind them? That is a core principle of mathematics, starting with basic assumptions and then using reasoning to determine more truths about specific objects or


Four-Color Proof

Interview with Dr. Sibley



Throughout my time researching the FTC , my thesis Advisor, Dr. Jeffery Sykes from Ouachita Baptist University was able to connect me with some great mathematicians. Namely, **Dr. Thomas Sibley** , Professor Emeritus of Mathematics from St. John's University in Minnesota. Dr. Sibley taught for well over 40 years at the collegiate level, but prior to that he taught high school mathematics in the Congo. Dr. Sibley has written many mathematical textbooks in various fields of math, namely in algebra, geometry, and logic. Dr. Sibley described himself as a **mathematical logic researcher** which was one of the main reasons I chose to interview him. He can help me gain insight into the **logic behind what proofs** are and how they are utilized in mathematics as a whole.

Hearing about the Proof of the FCT

In 1974, 2 years before the final Proof was published, Dr. Sibley said “I was over in the Congo, and one of my friends told him about these people that had come up with a counterexample of the four color theorem, but it ended up being an **April fools joke in 1974**.” However, when I asked him about the actual proof being published in 1976, he said It was a big deal in 1976, I was in graduate school and my advisor in logic told me that **computers in mathematics were a terrible idea**. How can we stoop so low as to trust a computer with our gorgeous mathematical proofs?”

Hearing about the Proof Continued.

Furthermore, Dr. Sibley noted that the first time he ever touched a computer was in 1982, 2 years after completing his PhD, where he typed his dissertation on a manual typewriter. Dr. Sibley also said that when Appel and Haken published the proof, he felt as if it could've been right or it could've been wrong. He mentioned that it was more a question of probability. There were so many cases to check, and probability wise, **it was probably correct**, but Dr. Sibley wasn't really convinced at the time.

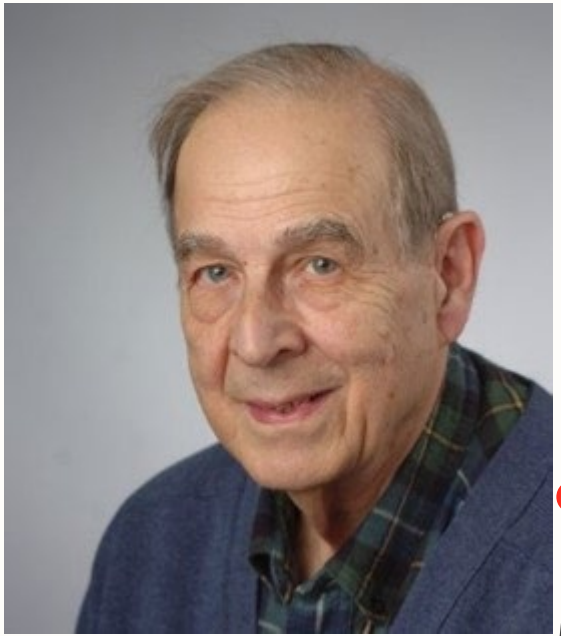
Dr. Sibley's final thoughts

In Dr. Sibley's concluding statement, he said that **by the 1980's,** many mathematicians were beginning to realize that the **skeleton of the proof was solid** and that in principle it should work (talking about the Four Color Theorem). During this time, other proofs were being published with the use of computers and that there was a **shift that was happening among mathematicians** to where computers were beginning to be **seen as friends, and not enemies.** In addition, Dr. Sibley mentioned these programs that were being published that could be used to check proofs in mathematics that were being done with computers. Like an HTML validator, but for mathematical proofs. These proof validators did require specific syntax, but the more they were developed, the more widespread they became.

My Thoughts

I think that this was the overall belief among older mathematicians at the time, as the idea of using technology in mathematics was foreign. Mathematicians are known for being stubborn in their ways and ideas, but at the time they didn’t want to have to change the very way they studied, wrote, and learned mathematics.

Interview with Dr. Graver



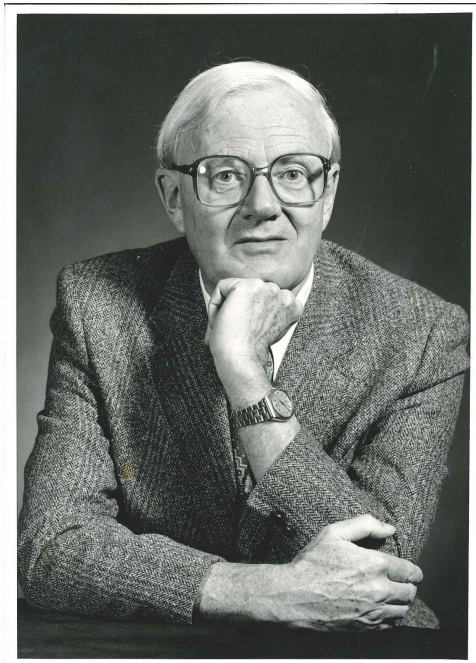
In addition to interviewing Dr. Sibley, I also had the opportunity to speak with Dr. Jack Graver. He began his studies at Miami University in Ohio, where he would earn his Bachelors in Mathematics. He then went on to the University of Indiana where he got his Masters and Doctoral Degree. From there, he taught at Dartmouth for a few years, and then at Syracuse where he would spend the rest of his career researching rigidity theory and the structure of large plane graphs. With over 35 publications, 15 articles, various NSF summer institutes, consulting for a university overseas, and even publishing his own book on rigidity theory. Dr. Graver would be a crucial resource for me in developing my own beliefs on the proof.

What is a Mathematical Proof?

I began our interview with the question about defining a proof in his terms. Dr. Graver's response varied in the sense that he defined a mathematical proof as a "logical argument that is accepted by the mathematical community." This was very interesting because Dr. Graver specifically mentioned the fact that it must be accepted by the mathematical community, which could've been biased based on our previous conversations leading up to the interview, but I do not wish to draw any hard conclusions.

Dr. Graver's Experience with the FCT

Dr. Graver mentioned how he was "at a conference at the University of Waterloo, where **Dr. William Tutte**, one of the most prolific graph theory researchers in the history of mathematics was leading this conference, and the minute that the announcement hit, the television stations from Toronto came all the way over to Waterloo to interview Dr. Tutte. The television reporters then put Dr. Tutte on the spot in front of all of these other mathematicians at the same conference, where Dr. Tutte would say that **he accepted their proof after seeing it**."



Dr. Graver's Experience Continued.

Dr. Graver's reaction was that because "Appel was a fabulous mathematician, and that Haken was a well known computer scientist **that I accept the proof** ." He even went on to say that mathematicians at the conference began arguing about the validity of the proof and how Dr. Tutte was incorrect in saying that it was a valid proof. In the same manner, others on Dr. Tutte's side believed the proof to be valid. It's almost like there was a giant **boxing match** over this proof, but thankfully it wasn't as severe as Hippasus' case.

Why Dr. Graver trusted the Proof

Dr. Graver decided to try and use computers for himself on a project he had already completed. He was researching something called a ramsey number, which was somewhat similar to the Four Color Theorem's kempe chains, and he decided to use a programming language called **Basic**. He was eventually able to learn and solve his earlier research problem in Basic, and so he then trusted the logic behind computers because he experienced them working for himself.

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Conclusion

Remarks

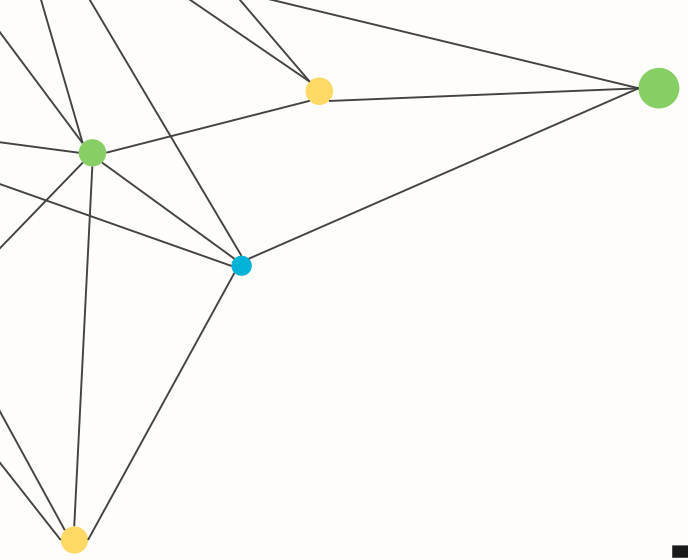
The Four Color Theorem’s proof may have caused a lot of havoc in the math community, but its implications have been great. Dr. Sibley wasn’t necessarily sure as to the validity of the proof, yet Dr. Graver trusted the opinion of Dr. Tutte. These two great mathematicians are able to have varying opinions as to the proof itself, yet both are able to form their own ideas and thoughts.

Improving on the Four Color Theorem

The Four Color Theorem was eventually reproved by reducing the number of uncountable sets down to under 640, which was originally over 1900, while also improving the code that was used to check each case. All of these meant more efficient programs and a much more elegant and simpler proof, yet still just under 640 cases to check.

Final Conclusions

All in all, the Four Color Theorem has been quite the groundbreaker for using computers in proofs, starting some serious mathematical tension, and also a historical problem that continues to amaze those who study it.



Thank You!