1969

Mathematical Philosophy

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MATHEMATICAL PHILOSOPHY

A Report Presented to
the Department of Mathematics
Ouachita Baptist University

In Fulfillment of
the Requirements for
Honors Special Studies H490

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by
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Spring 1969

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The purpose of Mathematical Philosophy by Cassius J. Keyser is to delve into some of the more essential and significant relations between mathematics and philosophy. To see this relation, one must gain insight into the nature of mathematics as a distinctive type of thought. The standard of excellence in the quality of thinking to which mathematicians are accustomed is called "logical rigor;" clarity and precision are essentials. The demands of logic, however, cannot be fully satisfied even in mathematics, but it meets the requirements much more nearly than any other discipline. Thus, the amount of mathematical training essential to education is the amount necessary to give one a fair understanding of the strict logical standards involved. Mathematics, which is identical to logic, therefore, is not completely separate from philosophy but is, strictly speaking, one of its principle divisions. Whenever a mathematician compares two branches of mathematics, his attitude becomes that of the philosopher. The student of philosophy, on the other hand, needs to acquire that knowledge of mathematics which will bring him into sympathy with it.

The philosophy of mathematics, or the study of its foundations, is connected quite closely with the concept of the "postulate system." A "postulate" is any proposition taken for granted in a given branch, such as in Hilbert's geometry. A "postulate system", therefore, serves as an ideal prototype for guidance in rational exercises. For a postulate system to be consistent, its functions must not involve contradictions among themselves. All postulate sytems have certain aspects in common. Propositional functions are necessary: these are statements containing one or more real variables, real variable meaning a symbol whose meaning is undetermined in the statement, but to which one assigns values. A propositional function is a matrix of the propositions derived from it by substitution and has the same structure as the propositions it yields. By substitutuing some values for the variables, the function holds true, while for other values it is false.

All valid proof depends entirely upon the form of the premises, or postulate, and not upon any specific meanings assigned to their variables. When a specific subject matter is assigned to a group of propositions, a "doctrine" results. A "doctrinal function" is a body of logically related propositional functions having doctrines for its values. A given doctrinal function is "interpreted" whenever one derives from it one of its values, or doctrines. Because the doctrine, whether true or false, matches the doctrinal function, and because the statements composing the doctrine and corresponding statements of the doctrinal function are identical in form, it can be said that the doctrine and the function are like in form. Being like in form, the doctrines are discriminated among themselves only by contents. This difference of meaning of a true doctrine's content is not logical, but is purely psychological. Two postulate systems are equivalent or non-equivalent if the corresponding doctrinal functions are identical or non-identical.

In inventing a postulate system, the inventor is almost never aiming at the establishment of what has been called a doctrinal function. He is aiming at establishing as autonomous a particular one of the many doctrines which a doctrinal function has for its values. How can one be certain that he has found a set of verifiers for a given collection of propositional functions? The answer is, "One can never be absolutely certain."

The doctrinal function is a branch of pure mathematics, while the doctrine is a branch of applied mathematics. Bertrand Russell says that "mathematics is the science in which one never knows what one is talking about nor whether what one says is true." When a doctrine reaches the "ideal" only then does it become mathematical; the immense majority of doctrines are, then, non-mathematical, lacking autonomy. However, we seem to be much less concerned in having our doctrines ultimately true than to have them instantly effective.

The mathematical idea denoted by the term transformation implies a change in form, but not in value. The "law" of a transformation is any rule which, given any of the elements dealt with, determines its transform. An example of a true mathematical one-to-one transformation is the process of counting objects. In dealing with the static world of unchanging things, concepts, one deals with the dynamic world of changing things by logical thought. The idea of invariance or of permanence in the midst of change, is as old as man's dream of eternal things. The theory of mathematical invariance began in the 1770's with Lagrange's obervations about the discriminants of the quadratic expression $ax^2+2bxy+cy^2$. A law of nature, thus, is simply an invariant relation among variant terms. In this conception of natural law and the consequent conception of natural science as having for its aim discovery of invariant relations among the things that appear and disappear in the world, there is nothing new except for their outward appearance. The main endeavor of the philosopher is to find the ideal or the invariant; for the mathematician the theory of invariance always acts as a guide.