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Fostering Geometric Thinking

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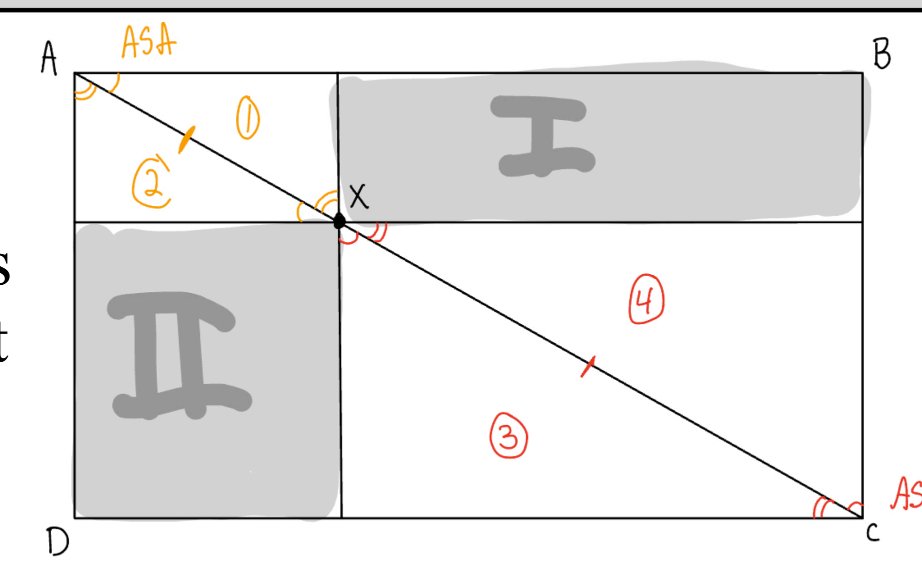
Introduction

Fostering Geometric Thinking is a learning process that guides critical thinkers in the problem solving process. In the book "Fostering Geometric Thinking," by Mark Driscoll, Driscoll describes four different geometric habits of mind that are essential to Fostering Geometric Thinking. These four geometric habits of mind are as follows :

1. Reasoning with Relationships
2. Investigation Invariants
3. Generalizing Geometric Ideas
4. Balancing Exploration and Reflection

2 Investigating Invariants

The next Geometric Habit of Mind is Investigating Invariants. An *invariant* is usually something about a situation that stays the same even as parts of the situation change. To demonstrate this see the figure at right. Our objective is to prove that the area shown at right is always equal regardless of where a point along a diagonal is placed.



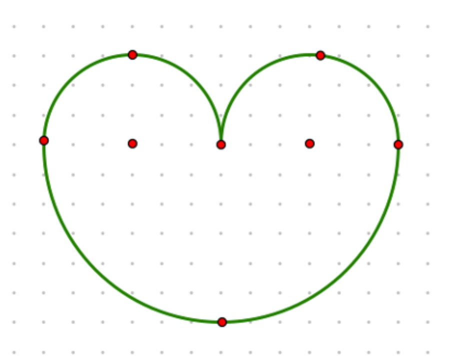
Area of ▲ ABC = Area (1) + Area (4) + Area (I)
Area of ▲ ADC = Area (2) + Area (3) + Area (II)
But, through ASA congruence theorem, Area (1) = Area (2), and Area (3) = Area (4).
Now, Area of whole rectangle:
▲ ABC = ▲ ADC.
Then through Algebra, we find that area (I) = area (II).

In conclusion, throughout this activity we explored the *invariants* in this specific problem. While our point X was sliding up and down the diagonal, the areas of the two regions remained the same. This is quite surprising and something that you definitely wouldn't expect!

1 Reasoning with Relationships

Reasoning with Relationships is the first big Geometric Habit of mind described by Driscoll. In this section, we explore questions like "how are these figures similar/ different," and "what else here fit's this description?" To illustrate this, imagine you are in a rural country called round county. This county is full of pastures, but the mayor decided that the pastures can only be made up of circles and partial circles. An example of three different pastures is shown below.

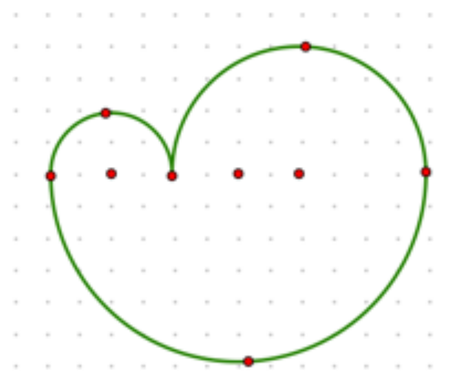
Pasture A



$Circumference\,(A) =$
$= \pi(6) + \pi(3) + \pi(3)$
$= \pi(6+3+3)$
$= 12\pi$

$Area\,(A) =$
$= \frac{\pi(6)^2}{2} + \frac{\pi(3)^2}{2} \cdot 2$
$= 27\pi.$

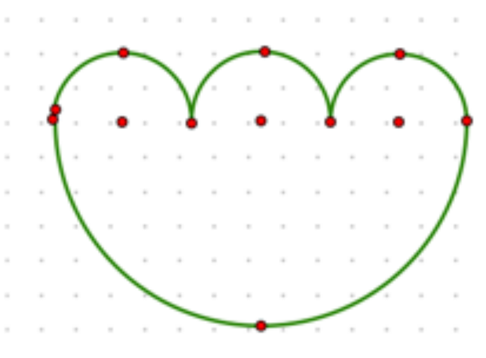
Pasture B



$Circumferencc\,(B) =$
$= \pi(6) + \pi(2) + \pi(4)$
$= \pi(6+2+4)$
$= 12\pi.$

$Area\,(B) =$
$= \frac{\pi(6)^2}{2} + \frac{\pi(4)^2}{2} + \frac{\pi(2)^2}{2}$
$= 28\pi.$

Pasture C

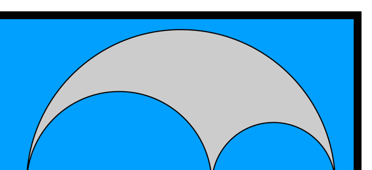


$Circumfererence\,(C) =$
$= \pi(6) + \pi(2) \cdot 3$
$= 12\pi.$

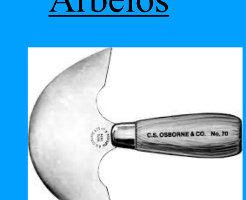
$Area\,(C) =$
$= \frac{\pi(6)^2}{2} + \frac{\pi(2)^2}{2} \cdot 3$
$= 24\pi.$

In conclusion, we have shown, while all of the perimeters of the different pastures are the exact same, their areas differ.We were able to use the relationships between the area and perimeter to find out the largest area given our requirements. Furthermore, we were able to take advantage of the distributive property in each of the different scenarios when finding the pastures respective circumferences. Reasoning with relationships is key in Fostering Geometric Thinking.

Diving Deeper- consider the *Arbelos*, which is very similar to the pastures described above. The *Arbelos* is formed from drawing 2 semi-circles, and then a third circle with diameter of the 2 semicircles combined. The *Arbelos* is commonly used by shoemakers across the world because of its effectiveness in cutting leather.



Arbelos

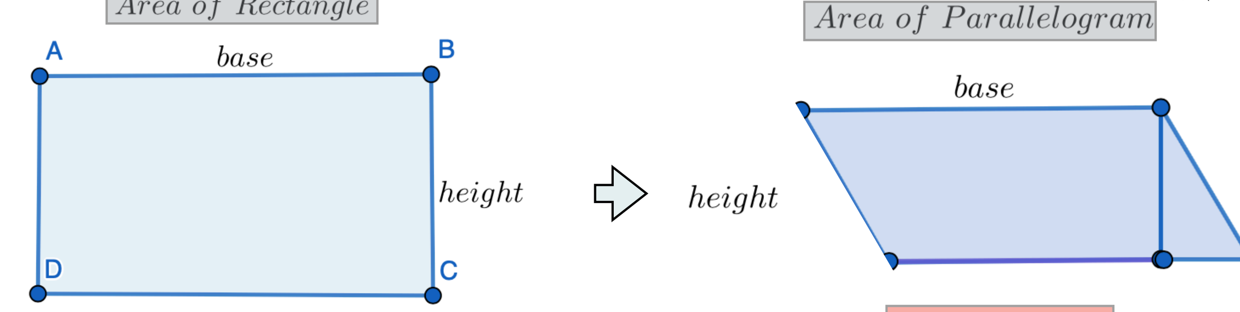


Shoemaker's Knife

3 Generalizing Geometric Ideas

Generalizing Geometric Ideas is a staple of mathematics. Finding certain patterns that hold true are vital in the problem solving process. In this section, we derive the area of common polygons utilizing the known formula of the area of a rectangle.

Area of Rectangle

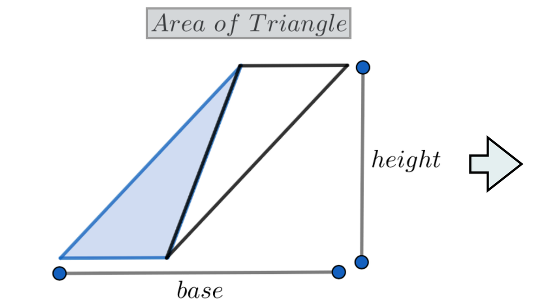


$= base * height$

Area of Parallelogram

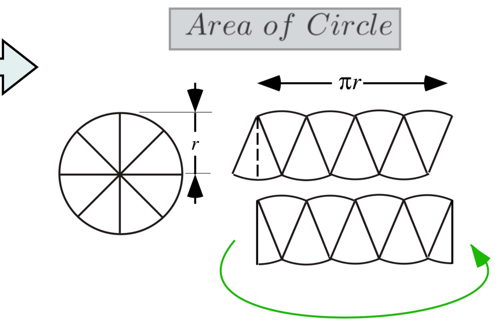
$= base * height$

Area of Triangle



$Area\ of\ Triangle = 1/2 * Area\ of\ Parallelogram$
$= 1/2 * base * height$

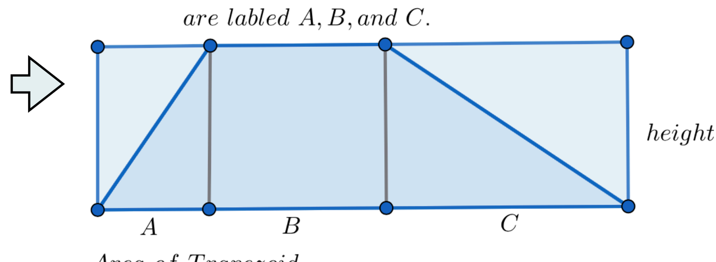
Area of Circle



$= \pi r^2.$

Area of Trapezoid

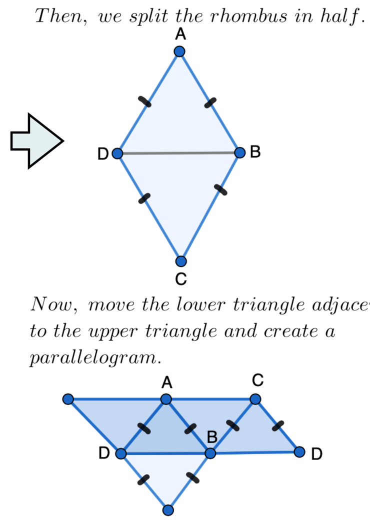
Suppose the bases of each rectangle are labled A, B, and C.



Area of Trapezoid
$= (A+B+C)(height) - \frac{(A * height)}{2} - \frac{(C * height)}{2}$
$= \frac{1}{2} * height * (2A + 2B + 2C - A - C)$
$= \frac{1}{2} * height(A + 2B + C)$
$= \frac{1}{2} * height(sum\ of\ bases).$

Area of Rhombus

We begin with rhombus ABCD. Then, we split the rhombus in half.



Now, move the lower triangle adjacent to the upper triangle and create a parallelogram.

$Area\ of\ Rhombus = 1/2 * area\ of\ Parallelogram$
$= 1/2 * base * height$

4 Balancing Exploration and Reflection

The last Geometric Habit of Mind if Balancing Exploration and Reflection. We take a step back and ask the "What if" types of questions and seeing what happens. Curiosity is peaked in this phase of the learning process and often leads to really cool research topics! Specifically, the Möbius strip is an extremely unique topological figure with interesting properties. So let's explore this strange side of mathematics.

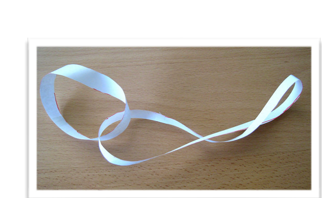
Möbius Strip

One-Sided surface with no boundaries! I was able to start drawing a line at any arbitrary point down the middle of the paper. The line I drew was continuous and was conceptually very interesting.

Möbius Strip cut in half

After cutting down the central line in the first image, I was left with a more twisted Möbius Strip that was skinnier, yet the same width around the entire strip!

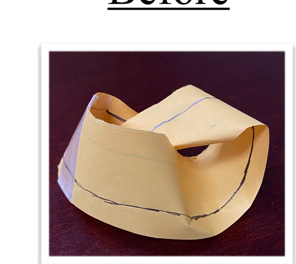
Möbius Strip cut in thirds



After cutting down a line centered along the one-third mark of the original Möbius Strip, we were left with an interlocked chain with one that was much skinnier in width than the other. The other strip was thicker and resembled our original Möbius Strip.

My Experiment:
Investigating a Möbius Strip that was folded three times, and then cut down a line centered along a point at the one third mark.

Before



After

After cutting my triple folded Möbius Strip down the one third line I was left with an even weirder linked Möbius Strip with one longer skinny piece as well as another wider piece that was tied up like a bow.

Diving Deeper- we are able to calculate the Euler Characteristic for different geometric figures using a formula that is as follows:
$\chi = V - E + F.$
We can utilize this to determine the Euler Characteristic of our Möbius Strip.
$= 0 - 2 + 2$
$= 0.$

Thus, because the Euler Characteristic of the Möbius Strip is 0, that means that any of the following shapes below are topologically similar to the Möbius Strip, and can be morphed into one another.