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A Special Studies Paper on Mathematics in Computer Programing

Linda Gamble
Ouachita Baptist University

A SPECIAL STUDIES PAPER ON

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by

Linda Gamble

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The profession of programing has existed for two decades or longer. Computer programing and the organization of computer analysis have gone beyond the reach of careful clerks with high school education. There is a growing demand for persons who are competent researchers, experienced in the use of multivariate methods, and skilled as computer users and programers. Mathematics and engineering predominate. The programer should have a knowledge of mathematics and a familiarity with computer hardware and must understand the content of the problem to be solved.

The computer often assists in studing the operation of a particular type of business activity (such as production or transportation) in order to find ways of designing a new system for performance and economy, Computers have become a permanent part of the business operation because it can handle quickly and economically large amounts of routine data-processing. The computer can produce certain output data such as reorder quantities for inventory items or amounts of pay checks for individual employees by means of a well-defined model, that is a clear-cut set of logical rules or mathematic equations.

Computer systems that give the best or most economical solution to a business problem by manipulating a mathematical model of the business system are called optimizing systems.[1]

There are two main types of programing. The first is linear programing which is used by a wide variety of industries to solve their operation and expansion problems. Linear programing may be described as a mathematical procedure for picking the "best" weighted combination of alternatives when the system to be optimized is described by a set of linear algebraic restrictions and the effect of all the alternatives selected can be expressed as a linear function of the alternatives. The best solution would be the one giving least cost or most profit. Large operating systems involving multi-refinery operations and over-all inventory planning are being set up for computer solution. Some of the systems may reach 1000 equations.[2]

A second type of programing is dynamic programing, a mathematical tool which is being used to find solutions to complex business problems.

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[1]Irving J. Sattenger, Applying Computers, Howard Allen, Inc., Publishers, 1963.

[2]Ibid..

Dynamic programing can be used when there is no mathematical relationship between the parts of the probelm. A large oil company may use dynamic programing to determine the most economical loading of pumps in its pipe line system.[3]

What does the computer staff do with a problem presented to it? There is a particular procedure for programing a digital-computer solution. An engineering problem must usually be subjected to numerical analysis in order to reduce it to a form requiring operations which the computer can handle. The equations resulting from the numerical analysis must be used to draw a detailed flow diagram indicating the processing of the information within the computer.

A numerical analysis of a problem consists of converting the original problem statement to combinations of additions, subtractions, multiplications, divisions, and certain logical operations, since these are the operations the computer can perform.

The conversion of mathematical statements from one form to another consists of such steps as the following:

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[3] Ibid.

(1) An equation may be converted to a form in which the unknown quantity is expressed explicitly rather than inplicitly. If the quadratic equation $ax^2 + bx + c = 0$ is solved for x by means of a digital computer, the equation must be reworked to the form

$$x = \frac{-b \pm b^2 - 4ac}{2a}$$

(2) Differential equations may be converted to a difference equation to permit the digital computer to solve them by means of fundamental arithmetic processes. A differential equation of the form

$$\frac{dt_a}{dt} = -K\,(T_a - T_e)$$

is converted to a difference equation of the form

$$\frac{T_{a(i+1)} - T_{a(i)}}{t_{i+1} - t_i} = -K\,(T_{a(i)} - T_{e(i)})$$

where $T_a(i+1)$ is the value of T_a at $t + t_{(i+1)}$, and $T_{a(i)}$ are the values of T_a and T_e at $t = t_i$.

(3) Equations in which trigonometric or logarithmic functions of a variable are required may be programed so as to obtain these functions by the solution of power series of the variables

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots + (-1)^{n+1}\frac{\theta^{2n-1}}{(2n-1)!}$$ [4]

[4]*Ibid.*, pg. 122.

To get a computer going on the solution of a problem, the programer must do more than convert the original problem as was just discussed. Each step or instruction for the solution of a problem must be provided for the computer in detail. Programers, the people who prepare the sets of instructions usually do four things:

(1) they analyze the problem carefully and break it down into parts

(2) they prepare a plan, called a flow chart, for solving the problem

(3) they must put the individual steps indicated by the flow chart into a form which the computer has been built to handle

(4) they will test the set of steps on sample data to check for errors.[5]

One type of problem solved easily by computer would be the calculation of an arithmetic mean and standard deviation. The formulas selected are those designed for use with the original, ungrouped data.

$$M = \frac{\Sigma X}{N}$$

$$S.D. = \frac{1}{N}\sqrt{N\Sigma X^2 - (\Sigma X)^2}$$

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[5]Vincent S. Darnowski, Computers - Theory and Uses, 1964.

$$\text{Variance} - (SD)^2 = \frac{N\Sigma X^2 - (\Sigma X)^2}{N^2}$$

Program design involves the analysis of the mathematical formulas in terms of the computer operations to be performed. The flow chart represents the movement of the data through the computer. An example of the flow chart for this problem is shown on the next page.[6]

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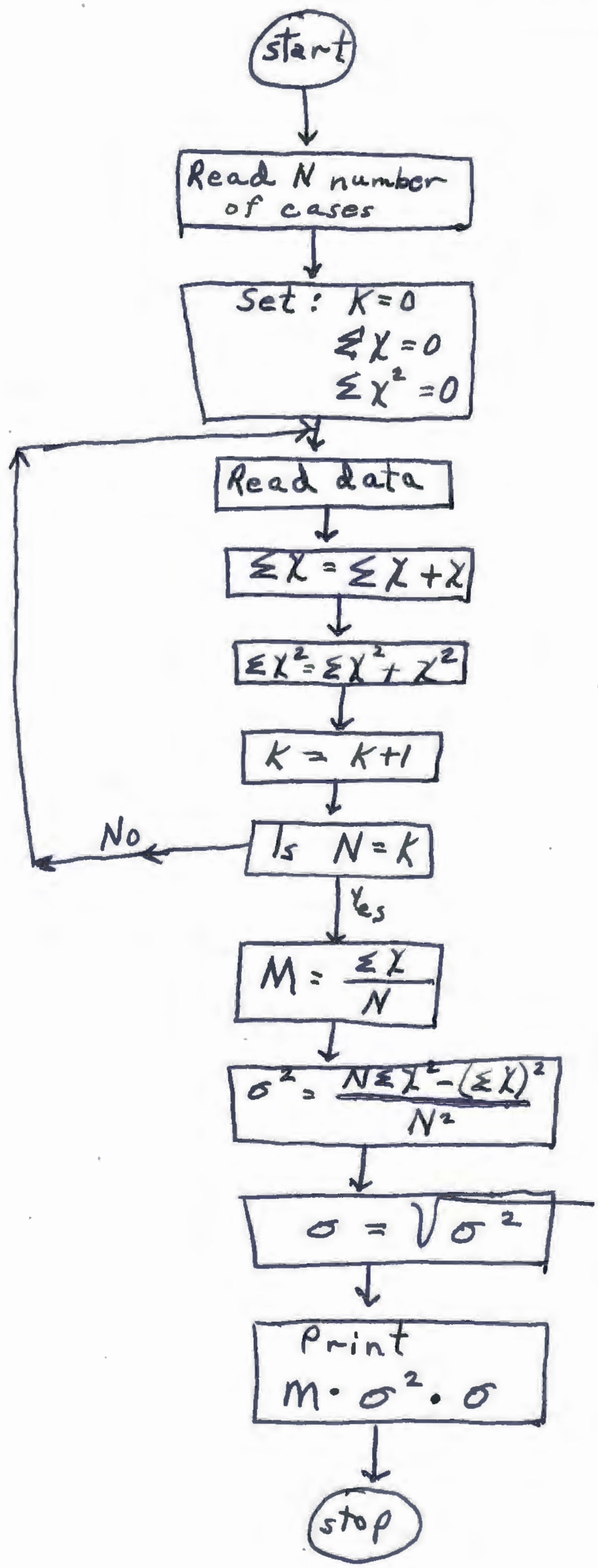
[6]Harold Borko, Computer Applications in the Behavioral Sciences, Prentice-Hall, Inc., Englewool Cliffs, N. J., 1962.

Flow Chart: Arithmetic mean and standard deviation



standard deviation σ

A very simple example of a problem that could be presented to a computer would be:

A man has three keys on a ring all different sizes. The car key is smallest. The problem is programing the computer to select from the three keys the one that fits the car. There are several possibilities to put into the computer. One is to put into the computer the size characteristics of the car key, use this as a standard, and compare all given keys to this standard. The key that matches would be the one. Another possibility is to compare two of the three keys and eliminate the larger, compare the remaining two and eliminate the larger, and thus the smallest is left. At this point the mathematician programer is asked to help decide which method would be most efficient. The criteria for efficiency in this case are simplified coding, reduced execution time, and minimized storage facilities. The mathematician makes a mathematical model for each possibility and selects the simpliest. The mathematician must then writes a set of equations for the solution. Letting A, B, and C equal the three keys the equations would be:

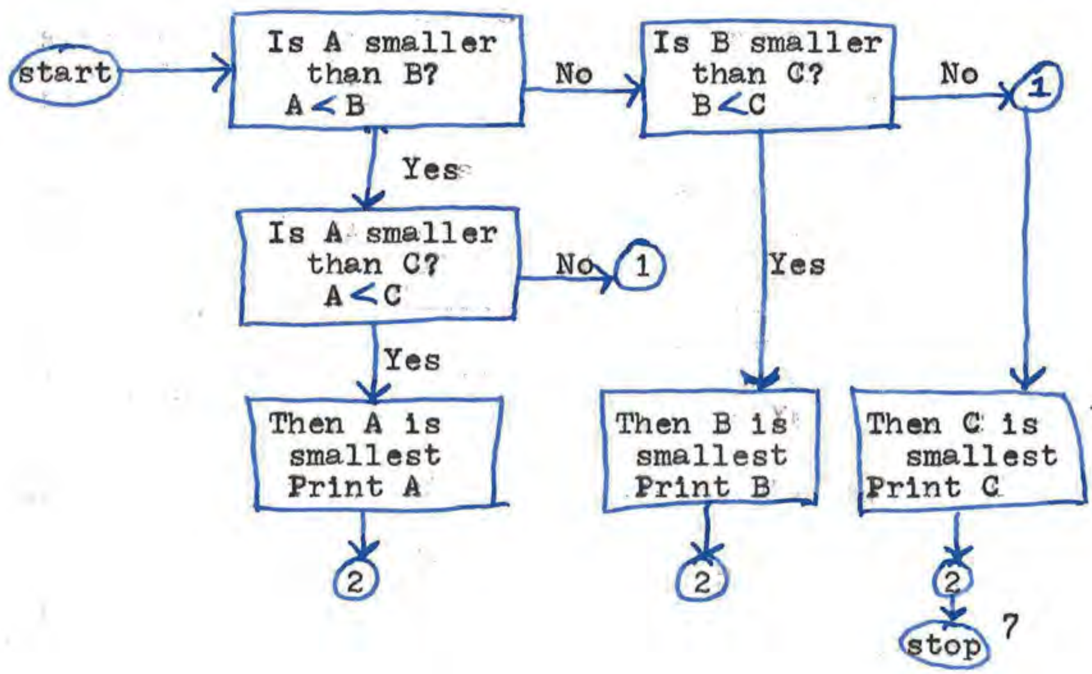
(1) $A < B$, $A < C$ $\therefore$ A is smallest

(2) $A < B$, $C < A$ $\therefore$ C is smallest

(3) $B < A$, $C < B$ $\therefore$ C is smallest

(4) $B < A$, $B < C$ $\therefore$ B is smallest

The systematic analysis of the problem ends with the writing of the mathematical model of the problem and its solution. The program designer now draws the flow chart which would look like this:



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The coder then writes the program that will enable the computer to solve to problem.

Although these are only simple problems, a computer is put to a much harder test. When an astronaut blasts into space, every

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[7] *Ibid.*

action of the space vehicle during launch is directed by computers. Every phase of the manned flight is sent to the computation center. Signals telling of fuel conditions inside the capsule are all possessed by computers within 1½ seconds of transmission. The processing of airline reservation are sometimes done by computers. Computer systems take in information on all known flight plans of the United States. If an aircraft approaches the United States comparisons between this data and flight plans are made. Equations describing the motion of the earth's atmosphere can be simlified and reduced by computers. These are just a few of the vast numbers of uses of computers.[8]

Mechanical calculators and the early electronic computers calculate in the decimal (normal base 10) system. To simplily computer circuitry, the "binary system" is used in computers. This is base 2. This cuts out more work for the mathematicians. There are only two numbers used in this system and that is 0 and 1. Therefore, the number 18 would be written as 10010. It is important for the

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[8]Darnowski, loc. cit.

mathematician to be acquainted with the conversion of base 10 numbers to "binary" and "binary" numbers to decimal.

The conversion of the binary number 1101.01 to base 10 would be as follows;

$$\begin{array}{r}
2^3 \times 1 = 8 \\
2^2 \times 1 = 4 \\
2^1 \times 0 = 0 \\
2^0 \times 1 = 1 \\
2^{-1} \times 0 = 0 \\
2^{-2} \times 1 - .25 \\
\hline
13.25
\end{array}$$

Conversely, a decimal number may be converted into its binary equivalent by successful subtracting the powers of 2 and/or by dividing by the powers of 2. To convert the decimal number 106 by the first method:

$$\begin{array}{lcl}
 & \text{remainders} & \\
2 \underline{|106} & 0 & \\
\quad 2 \underline{|53} & 1 & \\
\quad\quad 2 \underline{|26} & 0 & \\
\quad\quad\quad 2 \underline{|13} & 1 & = 1101010 \\
\quad\quad\quad\quad 2 \underline{|6} & 0 & \\
\quad\quad\quad\quad\quad 2 \underline{|3} & 1 & \\
\quad\quad\quad\quad\quad\quad 2 \underline{|1} & 1 & \\
\quad\quad\quad\quad\quad\quad\quad 0 & &
\end{array}$$

The second method:

$$\begin{array}{r}
1 \\
2^6 \overline{)106} \\
\underline{64}
\end{array}$$

$$\begin{array}{r l} & 1 \\ 2^5 & \overline{)\,42} \\ & \ \ \underline{32} \end{array}$$

$$\begin{array}{r l} & 0 \\ 2^4 & \overline{)\,10} \end{array}$$

$$\begin{array}{r l} & 1 \\ 2^3 & \overline{)\,10} \\ & \ \ \ \underline{8} \end{array}$$

$$\begin{array}{r l} & 0 \\ 2^2 & \overline{)\,2} \end{array}$$

$$\begin{array}{r l} & 1 \\ 2^1 & \overline{)\,2} \\ & \ \ \underline{2} \end{array}$$

$$= 1101010$$

$$\begin{array}{r l} & 0 \\ 2^0 & \overline{)\,0} \end{array}$$

The designers of computer programing didn't stop at this. The octal system provides a method for reducing the number of binary bits to 1/3 while still maintaining all the advantages of that system. The octal system is based on base 8. Therefore the number 8 is expressed as 10. Each octal digit has its three-bit binary equivalent. Convert octal to binary by this method:

number 367 base 8 to binary;

3 = 011

6 = 110

7 = 111 = 011 110 111

Convert binary 100 101 001 to octal:

100 = 4 101 = 5 001 = 1 equals 451

Also, it is possible to convert octal to decimal by writing it as the sum of its powers of 8. Therefore 374 octal would be: [9]

$$8^2 \times 3 = 192$$
$$8^1 \times 7 = 56$$
$$8^0 \times 4 = \underline{4}$$
$$252$$

Today's Space Age opens a wide field for the mathematician, chemist, physicist, and the computer. Predicted decay in the lunar gravitational field might be confirmed by precisely measuring changes in the radial distance between the earth and moon. Computers with their time interval plug-in can resolve the time it takes signals to bounce off the moon's surface some quarter of a million miles away and convert this time to distance - feet or inches - out to ten or eleven significant figures. The counter itself can decide from the measurement how many figures are significant.[10]

At the same time computers are being used on such an advanced level, effort is being made to put the computer to use in the secondary school level. Project "Local" was carried out

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[9] Borko, *loc. cit.*

[10] *Scientific American*, vol. 221, number 3, September, 1969, pg. 33.

in some northern schools to demonstrate and evaluate the use of the computer in secondary mathematics. Workshops are provided to train teachers to use the computer in existing courses and as teaching aids. They include problems designed for drill, review and testing, labortory experiments, problem solving, and ideas for operating with mathematical relationships to science.[11] Computers may prove to be very successful in school and educational situations.

In the present decade and those to come mathematicians will be in continuous demand whether in the field of computer programing or in other fields in which a knowledge of computer programing is necessary. Many industrial companies are training their own personnel in computer work. Many school districts are making efforts to use computers in secondary mathematic classes. Space centers use computers continuously during the space flights and mathematicians are called day and night. After a study of computer programing a mathematician will see that this field requires much more of a person than just pushing buttons and pulling levers. Someone has to calculate what will be feed to the computer and interpret the results.

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[11] Robert Slagle, Computer-Assisted Learning, The Scientific Teacher, vol. 36 (Jan. 1969), pg. 60.

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