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An Introduction to the Standard Model and the Electroweak Force with a Numerical Analysis of the Yang-Mills-Higgs Equation

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An Introduction to the Standard Model and the Electroweak Force With a Numerical Study of the Yang-Mills-Higgs Equations

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Outline

Introduction

Particle physics is a relatively new field in science that has made amazing discoveries in the last few years. Particle physics seeks to know the most basic structure of mass and force which makes up our universe. This search has made tremendous leaps forward in the last part of this century with the use of particle accelerators and theoretical advances. The work of particle physics has begun to accumulate to the formation of what is known as the standard model. This is a model of the universe which includes all basic forces and matter. Though this model is not yet complete, it is generally accepted. One of the most sought after missing links in this model is that of the Higgs boson. Theoretically it is well understood, but has yet to be physically observed. These theoretical ideas give rise to the Yang-Mills-Higgs equations which govern the Higgs boson. These equations exhibit an order-to-chaos transition which is very interesting.

Standard Model

In 1911 Ernest Rutherford discovered the nucleus of the atom. Within the next 81 years the number of particles in the universe that had been observed multiplied by over 3000%. The beginning of the standard model was the discovery of the first generation of subatomic particles. These subatomic particles that make up protons and neutrons (and other particles) are known as quarks. The first of these quarks were the "up" and the "down" (abbreviated "u" and "d"). The up quark has a charge of +2/3 and the down has the charge of -1/3. Three quarks are required to make up a proton or a neutron. As one can see, if one combines two up quarks and one down quark, one is left with a grouping of charge +1, which is the charge of the proton. The electron, which is in the first generation, had been known about for quite a while. This gives us three particles in the first generation, but it is not complete. The neutron has been seen to "decay" into an electron (or beta particle) and a proton. This beta decay should produce an electron traveling in one direction, with a proton traveling in a direction parallel to the electron in the opposite direction. Experimentation has shown, however, that this is not the case. Due to the law of conservation of momentum, the neutrino was theorized. The neutrino (ν) is a particle which has no electric charge, and possibly no mass. The up and down quarks, electron, and the neutrino make up what is known as the first generation of the standard model. See table 1.

Table 1

u	d
ν	e^-

While the up and down quark were being discovered, another quark was discovered, the strange (s). It has a charge of -1/3, similar to the down quark, but a much greater mass. Another charged particle similar to the electron had been discovered, the muon (μ^-). It appeared to have the same charge as the electron, just a much greater mass (200 times that of the electron). This led particle physics to theorize the existence of two more particles, the charm (c) quark (charge +2/3) and the muon neutrino (ν_μ). These particle were later observed. These give us the second generation. See table 2.

Table 2

c	s
ν_μ	μ^-

Recently, another set of quarks and charged particles were observed. The first of these to be observed was the bottom (b) quark. Again, similar to the down and strange, only much more massive than both. This led physicists to theorize the existence of the top (t) quark. This very massive particle was only recently discovered. The electron-related particle is the tau (τ^-) and its associated neutrino the tau neutrino (ν_τ). This set completes the third generation. See table 3. Most particle physicists believe this is the last generation of subatomic particles to be discovered.

Table 3

t	b
ν_τ	τ^-

The above mentioned particles are all of a similar family called fermions. Fermions are characterized by having a spin=$\frac{1}{2}$.

The spin of a particle is an intrinsic property of it. In an effort to account for both fine structure in spectral lines and the anomalous Zeeman effects, Goudsmit and Uhlenbeck proposed in 1925 that the electron possesses an intrinsic angular momentum (spin) and associated with this angular momentum a certain magnetic moment.[1] This property of spin is an intrinsic part of all fundamental particles, and is a conserved quantity. (This is associated with conservation of angular momentum.) Spin-$\frac{1}{2}$ particles obey the Pauli exclusion principle. The Pauli exclusion principle prohibits two identical fermions from occupying the same state of energy and spin (in a close proximate location, or particle). This idea will become extremely important later.

Given these basic particles, we next describe the basic forces between these particles. This will give us a fundamental view of all interactions that occur. In the standard model and quantum field theory, all basic forces have associated fundamental particles which mediate or transport this force. These particles are called bosons. Bosons are integer spin particles. (Integer spin particles have a spin that is 1, 2, 3, ...) Since they are fermions, they not do not obey the Pauli exclusion principle.

The force with which we are most familiar, but which little is known about in quantum physics, is gravity. Subatomic particles experience this force, but on an insignificant scale compared with other fundamental forces. Gravity is theorized to be mediated by

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[1]Arthur Beiser, <u>Concepts of Modern Physics</u> (New York: McGraw-Hill) 238.

the graviton. This yet unobserved particle is spin-2 and interacts with all fundamental particles that have mass. Mass is sometimes referred to as being the gravity charge.[2]

The next most familiar force is the electromagnetic force. Maxwell's equations, which combine the electric and magnetic into one force, have been understood for centuries, but with the rise of Einstein's special relativity and quantum mechanics, these equations received another look. When applying the electromagnetic theory to atoms and smaller particles, the effects of quantum mechanics become important. Heisenberg's uncertainty principle showed that virtual photons can be emitted for a short time. Once a particle approaches the speed of light, its mass increases and its observed lifetime is increased. This combination of quantum mechanics and special relativity led Feynman, Tomonaga, and Schwinger to derive the first quantum field theory: quantum electrodynamics or QED.[3] This showed the photon, a spin-1 particle with zero mass, to be the mediating particle of the electromagnetic force. This force acts only upon electrically charged particles.

One of the basic forces which most are not familiar with is that of the nuclear strong force. This force binds the three quarks of the hadrons (particles composed of the aforementioned quarks, i.e. protons, neutrons, etc.) together. This force is explained by quantum chromodynamics or QCD, which is analogous to

[2]Frank Close, <u>The Cosmic Onion: Quarks and the Nature of the Universe</u> (New York: AIP Press, 1986) 159

[3]Peter Watkins, <u>Story of the W and Z</u> (Cambridge: Cambridge UP, 1986) 46

the QED. The QCD theorizes the existence of a spin-1 particle with zero mass called the gluon to mediate this force. Not only do quarks come in each of the aforementioned types or "flavors", each type comes in six colors. The Pauli exclusion principle prohibits two fermions from occupying the same state and spin. Spin-$\frac{1}{2}$ particles spin either "up" or "down" ($+\frac{1}{2},-\frac{1}{2}$). One hadron, the Ω^-, contains three strange quarks all of which appear to spin up (spin-$+\frac{1}{2}$).[4] How can this be? Quarks must possess some further property that enables them to be distinguishable from one another in the Ω^-. This property is color. Quarks can be red, antired, blue, antiblue, green, or antigreen. The strong force is the force between the colors. As in electromagnetism, like colors repel, unlike colors attract; but also colors and anticolors attract more strongly. Since only quarks have color, only quarks experience the strong force. Color is sometimes referred to as strong charge.

The final fundamental force of the standard model, and the one which this paper will deal with, is that of the nuclear weak. The nuclear weak force is responsible for nuclear β^- decay. It is relatively short-ranged and mediated by three bosons, the W^+, W^-, and Z^0. These bosons are all spin-1 and have a large mass.

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[4]Close, 89.

Table 4

The Standard Model

u	d	c	s	t	b
ν	e^-	ν_μ	μ^-	ν_τ	τ^-

Forces	Particle(s)	Spin
gravity	graviton	2
electromagnetic	photon (γ)	1
strong	gluons	1
weak	W^+, W^-, Z^o	1

Nuclear Weak

In 1896, Becqueral discovered three types of radiation.[5] One form of this radiation is beta decay. Beta decay consists of a neutron (n^o) decaying into an electron (a beta particle, β^-), a proton (p^+), and an antineutrino (ν_e).

$$n^o \rightarrow p^+ + \beta^- + \overline{\nu_e} \tag{1}$$

(Notice charge is conserved.) By 1970 it was evident that protons and neutrons are made up of quarks. The modern view of neutron decay is that of one of the neutron's down quarks ($d^{-1/3}$) decays into an up quark ($u^{+2/3}$), an electron (e^-), and an antineutrino (ν_e).

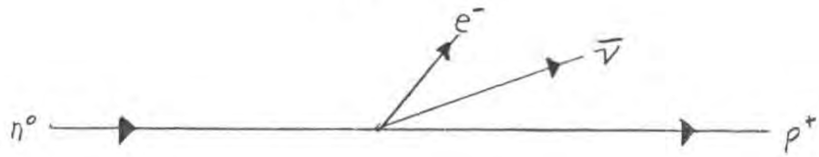
$$d^{-1/3} \rightarrow u^{+2/3} + \beta^- + \overline{\nu_e} \tag{2}$$

(Charge is still conserved. -1/3 = +2/3 - 1) In 1933, however, it was thought that protons and neutrons were without structure. It was by examining this beta decay Enrico Fermi developed the first

[5]Close, 27.

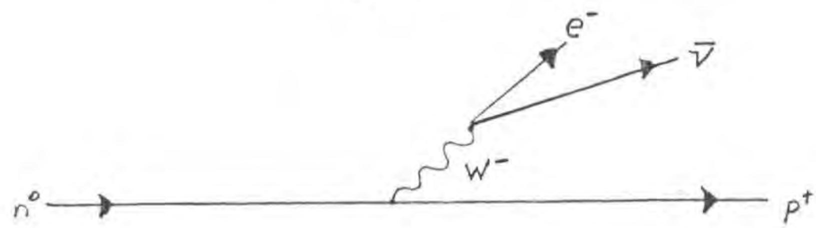
embryonic theory of the weak force.[6] Fermi's theory was inspired by QED where a neutron or proton absorbs a photon at a single point in space-time. Fermi proposed that the change of charge in the decay of a neutron into a proton is caused by the emission of an electron and an antineutrino at a point in space-time.[7] (See figure 1.)

Figure 1



In 1938 O. Klein suggested that a spin-1 particle ('W' boson) mediated the decay; this boson played the role in weak interactions like that of the photon in electromagnetism.[8] (See figure 2.)

Figure 2



Julian Schwinger, in 1957, noticed a problem in the spin-1 mediating particle of the weak force. It is massless, like the photon, and its interactions would have an infinite range in terms of distance like that of electromagnetism.[9] It was well known that

[6]Close, 106.

[7]Close, 107.

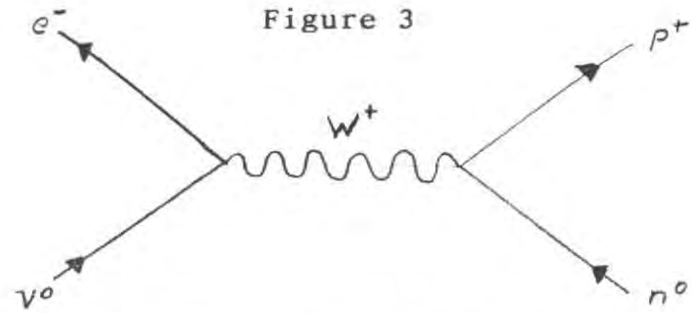
[8]Close, 107.

[9]Close, 107.

the weak force acts over a very limited range. To account for this discrepancy, Schwinger introduced the heavy W bosons as weak-force carriers. Schwinger also introduced the idea of two separate W's, one carrying a positive charge, the other negative: W^+, W^-.[10] An example of a positive current is that of a neutron and a neutrino interacting to produce a proton. (See equation 3 and figure 3.)

$$\nu^o + n^o \rightarrow e^- + p^+ \tag{3}$$

Figure 3



Schwinger also extended the analogy of the weak to the electromagnetic force to unify the models into one force. However, the analogy of weak to electromagnetic is not complete. The carriers of the weak force are short ranged and feeble while the carrier of the electromagnetic is stronger and has an infinite range. Schwinger realized that this difference in strength is subtle and to some extent illusory.[11] But, the problem of mass difference in force carriers persists. At this time another problem arose in the combining of electromagnetic and weak to a unified theory; symmetry laws of quantum mechanics requires four force carriers, and with the W^+, W^-, and γ (photon) there are only

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[10]Leon Lederman, The God Particle (New York: Houghton Mifflin Co., 1993) 329.

[11]Close, 107.

three. This symmetry requirement led to the postulating the Z^o boson of the weak force. Where the W^+, W^- carry charge, the Z^o carries no charge (just as γ). The transmission of the Z^o is known as neutral current and has been observed, though the Z^o itself has not been. Thus Z^o, along with W^+, W^-, and γ, keeps the required symmetry and so led theorists in a hunt for unification of electromagnetic and weak.

These theorists hopes lie in quantum field theory. Quantum field theory caused problems in that quantities that should be small and measurable appear in the equations as infinite. To counter this, Richard Feynman, et. al., introduced the idea of renormalization.[12] Renormalization involves "sweeping" up into formal expressions for quantities like physical mass and charge of a particle the infinities.[13] Yang and Mills in 1954 applied what is known as gauge symmetry to QED and generated electromagnetic forces, guaranteed the conservation of charge, and provided protection against infinities. Theories exhibiting gauge symmetry are renormalizable.[14] Yang and Mills ideas influenced theorists to force any electro-weak combined theory to have gauge symmetry.

Higgs Boson and the Higgs Field

Gauge symmetry, or gauge invariance, refers to "scale" (as in HO-gauge model railroad tracks). Gauge invariance implies that one

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[12] Lederman, 329.

[13] Anthony Hey, and Ian Aitchison, Gauge Theories in Particle Physics (Philadelphia: Adam Hilger, 1989), 168.

[14] Lederman, 330.

may change certain aspects of a field but not change the field itself. This can best be seen in the classical equations for static electromagnetic fields called Maxwell's equations. See equations 4 and 5.

$$\vec{B}=\nabla\otimes\vec{A} \tag{4}$$

$$\vec{E}=-\nabla V-\frac{\partial\vec{A}}{\partial t} \tag{5}$$

The electric field (E) is dependant upon the scaler potential field (V) and the vector potential field (A), while the magnetic field is dependant on the vector potential only.

Let us now change the scaler potential by equation 6.

$$V'=V+\alpha \tag{6}$$

In this equation α is a constant everywhere. Substituting V' for V in equation 5 gives

$$\vec{E}=-\nabla(V+\alpha) \tag{7}$$

This can be reduced to equation 8.

$$\vec{E}=-\nabla V-\nabla\alpha \tag{8}$$

But α is a constant. Therefore, equation 8 becomes

$$\vec{E}=-\nabla V \tag{9}$$

This equation is equation 2.

Now, we will change the scaler and vector potential fields as in equations 10 and 11.

$$\vec{A}' = \vec{A} + \nabla\chi \tag{10}$$

$$V' = V - \frac{\partial\chi}{\partial t} \tag{11}$$

Replacing A and V with these new values in equations 4 and 5 yields the following.

$$\vec{B} = \nabla\otimes(\vec{A} + \nabla\chi) \tag{12}$$

$$\vec{E} = -\nabla(V - \frac{\partial\chi}{\partial t}) - \frac{\partial(\vec{A} + \nabla\chi)}{\partial t} \tag{13}$$

These equations become equations 14 and 15.

$$\vec{B} = \nabla\otimes\vec{A} + \nabla\otimes(\nabla\chi) \tag{14}$$

$$\vec{E} = -\nabla V + \nabla\frac{\partial\chi}{\partial t} - \frac{\partial A}{\partial t} - \frac{\partial\nabla\chi}{\partial t} \tag{15}$$

Through further algebra we find the following equations.

$$\vec{B} = \nabla\otimes\vec{A} \tag{16}$$

$$\vec{E} = -\nabla V + \frac{\partial\vec{A}}{\partial t} \tag{17}$$

These equations are equivalent to equations 4 and 5. This is gauge invariance. We changed the potential fields drastically, but we did not change the electromagnetic fields. Changing the value of the electrostatic potential (V) by a constant amount (α) is an example of a global transformation (since the change in potential is the same everywhere).[15] We also expressed a local change in the electrostatic potential V (the $\partial\chi/\partial t$ term in equation 11). This term can be compensated for by a corresponding local change in the vector potential A (the $\nabla\chi$ in equation 10). Thus by including magnetic effects, the global invariance under a change in V by a constant can be extended to a local invariance, which is a much more restrictive condition to satisfy.[16] A combined electro-weak force must obey not only global invariance, but also the more restrictive local invariance of gauge. As the electro-weak theory exists, it does not meet gauge invariance requirements. Gauge symmetry, when applied to the electro-weak theory predicts four massless force carriers. Peter Higgs, of Edinburgn University solved this problem in 1964. His solution is known as spontaneous symmetry breaking.

Higgs, in a paper published in Physical Review Letters, presented this idea of symmetry breaking. Higgs proposed that a coupling of a gauge field (a field which obeys local and gauge symmetry) to another field could allow some spin-1 quanta of the

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[15] Aitchison and Hey, 46

[16] Aitchison and Hey, 46

gauge field to acquire mass.[17] This coupling does not require all quanta to acquire mass but allows some to.[18] This is spontaneous symmetry breaking, also referred to as the Higgs mechanism.

Higgs showed that if the gauge field of the electro-weak force was coupled to another field, the Higgs field, then the W^+, W^-, and Z^o could acquire mass, while allowing the photon to remain massless. This Higgs field has four degrees of freedom. Three of these degrees of freedom have been "eaten" to give the W^+, W^-, and Z^o mass. The remaining degree of freedom of the Higgs field corresponds to a quanta which is part of the Higgs field but not a part of giving mass to force carriers in the electro-weak field. This particle is called the Higgs boson.[19]

Dr. David Miller of London's University College gave the following analogy to Science Minister William Waldegrave to better explain the action of the Higgs field (for which he received a vintage bottle of champagne). Miller explains the Higgs field action by describing a cocktail party of political party workers. If the former United Kingdom prime minister Margaret Thacher were to walk into the room, she would immediately be surrounded by a cluster of people. As she moved through the room, she would attract the people she came close to, while those left in her wake would return to a more even spacing. If, say, a physics student

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[17]Peter Higgs, "Broken Symmetry and the Masses of Gauge Bosons." Physical Review Leters. 13 (1964): 508.

[18] Higgs, 1156

[19] Aitchison and Hey, 460

passed through the crowd, no one would group around and he/she would walk unnoticed. Now, if a juicy political rumor passed through the room some of the political busy bodies would flock to learn the details. They would then fan out to spread the gossip, each of them becoming the center of another clustering distinct from the kind inspired by Thatcher and would pass through the room.

In this analogy, the political party workers represent the Higgs field, Margaret Thatcher is one of the weak force bosons (W^+, W^-, or Z^o), The physics student is the photon, and the gossip is the Higgs boson. Like Thatcher, as the W^+ (or W^-, Z^o equivalently) moves through the Higgs field, it concentrates around the particle giving it mass. A photon moving through the Higgs field, like the student, goes unnoticed by the field and acquires no mass. Finally, like the gossip, the higgs boson is a cluster of the field without an associated boson.[20]

Higgs's theory of spontaneous symmetry breaking with a coupling to the Higgs field gives the required mass to the W^+, W^-, and Z^o bosons while the photon remains massless. Since the Higgs mechanism keeps the gauge symmetry, it must therefore be renormalizable. The field equations of the electro-weak theory coupled with the field equations of the Higgs field are quit complex and difficult to investigate analytically. Because of this, G t'Hooft studied these equations numerically. He showed that all of the infinities of the electroweak theory disappear with

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[20] "Good Booze and the Higgs Boson." <u>Science.</u> 261 (1993): 1392.

the action of the Higgs mechanism.[21] This result strongly suggests that the electroweak theory with spontaneous symmetry breaking is renormalizable.

The implications of spontaneous symmetry breaking and the Higgs field and boson goes beyond that of the electro-weak theory. According to current models, there was a rapid expansion of the universe in the first 10^{-35} seconds of its existence. After this time a phase transition occurred which heated the universe to a much higher temperature. During this period there was one basic force. After this heating the universe began to cool, and this single force separated into the many forces now present. This is known as the inflation model. This phase transition which caused the heating is due to the emergence of the Higgs field.[22] This phase transition due to the Higgs field is chaotic and this caused the heating. All of the universe contains the Higgs field. It is the same everywhere and permeates the vacuum. Particles influenced by this field acquire mass.[23] Theorists believe that the masses of the particles in the standard model, more directly all of the mass in the universe, are actually a measure of how strongly they are coupled to the Higgs field.[24] This inflation model suggests chaos in the early universe due to the coupling of the Higgs field

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[21] Leaderman, 331.

[22] Watkins, 230.

[23] Leaderman, 368.

[24] Leaderman, 368.

to gauge fields.[25]

Yang-Mills-Higgs Equations

The field of the electro-weak force is an SU(2)xU(1) gauge field. This is known more generally as a Yang-Mills field (YM). Dr. Tetsuji Kawabe of the Kyushu Institute of Design (Shiobaru, Japan) showed that YM fields that depend on both space and time are nonintegrable and intrinsically chaotic.[26] This suggests the inflation model. A more realistic model of the SU(2)Xu(1) gauge field of the electro-weak force is the space-time-dependent SU(2) Yang-Mills-Higgs (YMH) theory, which takes into account the coupling to the Higgs field.[27] This is a complex system, and difficult to investigate. If it is assumed, however, that both the gauge and Higgs fields are approximately uniform in space, then the YMH system reduces to classical rather than quantum mechanics. This system is known as the Yang-Mills-Higgs classical mechanics system (YMHCM).[28]

The YMHCM system of field equations yields the following set of coupled, second order ordinary differential equations (ODE's): (equations 18 and 19),

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[25] Tetsuji Kawabe, personal letter, July 31, 1996.

[26]Testsuji Kawabe, "Onset of chaos in time-dependent spherically symmetric SU(2) Yang-Mills theory," Physical Review D. (March 15, 1990), p. 1983.

[27] Tetsuji Kawabe, "Onset of chaos in SU(2) Yang-Mills-Higgs theory with monopole," Physics Letters B. (September 3, 1991), 399.

[28] Kawabe, Physics Letters B, 399.

$$\frac{d^2x}{dt^2}=-6y^2x-3x^3 \tag{18}$$

$$\frac{d^2y}{dt^2}=-6x^2y-6\kappa y^3+\kappa\nu^2y \tag{19}$$

where κ is the coupling constant, and ν the vacuum expectation value of the Higgs field. This system of ODE's is equivalent to a nonlinear mechanical system with hamiltonian as equation 20.[29]

$$H=\frac{1}{2}(p_x{}^2+p_y{}^2)+U(x,y) \tag{20}$$

$$U(x,y)=3x^2y^2+\frac{3}{4}x^4+\frac{3}{2}\kappa(y^2-\frac{1}{6}\nu^2)^2-\frac{1}{24}\kappa\nu^4$$

(p_x and p_y refer to the momentum in the x and y directions respectively.) Though this system of equations is much simpler than the field equations of YMH theory, it is still difficult to ascertain much analytically. Thus, these equations are best investigated numerically (numerically integrated). Many procedures are available for accomplishing this. Of the methods which have been devised, the Runge-Kutta method is the most general. This method can be applied to simultaneous differential equations, as in our case, as well as to single equations of any order, and is therefore quite adequate.

Numerical Technique

The fundamental problem in the numerical solution of ODE's is the solution of the first-order equation (see equation 22),

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[29] Kawabe, Physics Letters B, 400.

$$\frac{dy}{dx}=f(x,y) \tag{22}$$

subject to the initial conditions $y=y_0$ when $x=x_0$. We do not expect to find y as an explicit function of x. The objective is to obtain approximations of the values of the solution y(x) on a specified set of x values (x_1, x_2, x_3, ...). The first step in the point-wise solution of the initial-value problem is to approximate the value of y at equation 23.

$$x_1=x_o+dx \tag{23}$$

That is, as in equation 24.

$$y(x_1)=y_o+\Delta y \tag{24}$$

The simplest way to do this is to approximate Δy by equation 25.

$$\Delta y=dy=f(x,y)\,dx \tag{25}$$

Thus, from equations 24 and 25 we find the following.

$$y(x_1)=y_o+f(x_o,y_o)\,dx \tag{26}$$

This procedure can be repeated as far as required. This process is known as Euler's method. This method is unstable and inaccurate for many later steps because it is unsymmetrical. It advances the solution through an interval dx, but uses derivative information only at the beginning of that interval.[30] See figure 4.

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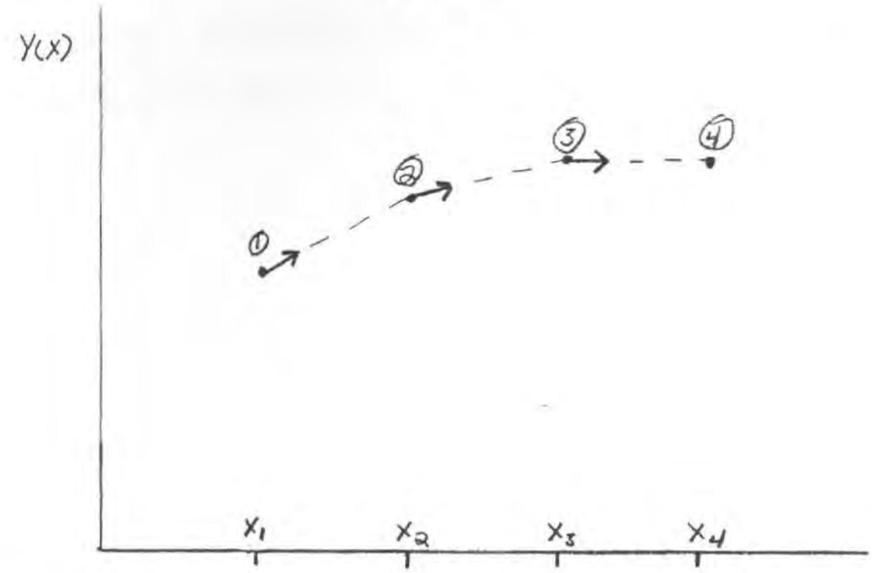
[30] William H. Press, et. al., Numerical Recipes (Cambridge: Cambridge UP) 704.

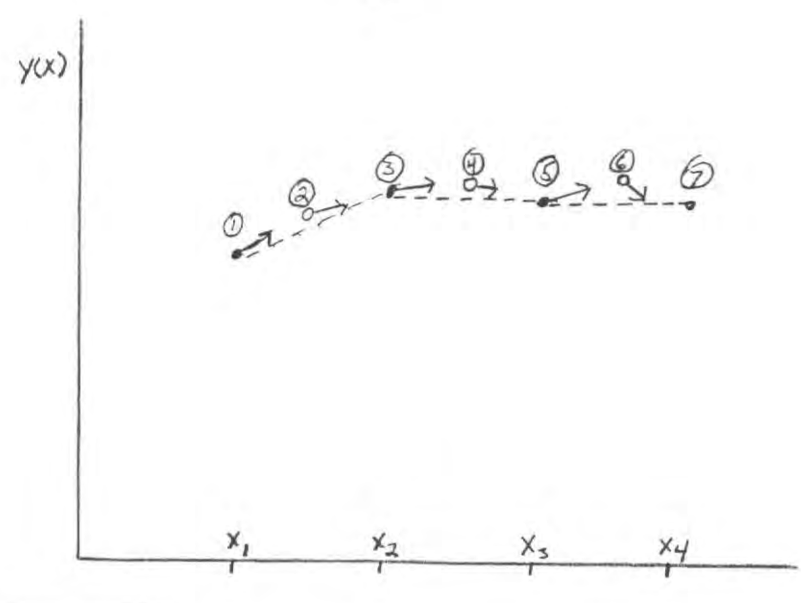
Figure 4



That means that the step error is only one power of dx smaller than the correction, i.e. a term of order $(dx)^2$ added to equation 26.[30]

Consider the use of a step like equation 26 to take a "trial" step to the midpoint to compute the "real" step across the whole interval dx. Figure 5 illustrates this.

Figure 5



[30] Press, 704.

In the figure, filled dots represent final function values, while open dots represent function values that are discarded once their derivatives have been calculated and used. This can be represented by equations 27.

$$k_1 = f(x_n, y_n)\, dx \tag{27}$$

$$k_2 = f(x_n + \frac{1}{2}dx, y_n + \frac{1}{2}k_1)$$

$$y_{n+1} = y_n + k_2 + O(dx^3)$$

As indicated by $O(dx^3)$, this method has an error term on the order of dx^3. This is a second-order method known as the midpoint method.[32]

The fourth-order Runge-Kutta method is very similar to this method. It utilizes several midpoint method style steps and combines them using a match to a Taylor series expansion up to a fifth-order term of the step size ($O(dx^5)$).[33] The classical form of the fourth-order Runga-Kutta formula is given by equations 28.

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[32]Press, 704.

[33]Press, 702.

$$k_1=f(x_n,y_n)\,dx \tag{28}$$

$$k_2=f(x_n+\frac{dx}{2},y_n+\frac{k_1}{2})\,dx$$

$$k_3=f(x_n+\frac{dx}{2},y_n+\frac{k_2}{2})$$

$$k_4=f(x_n+dx,y_n+k_3)\,dx$$

$$y_{n+1}=y_n+\frac{k_1}{6}+\frac{k_2}{3}+\frac{k_3}{3}+\frac{k_4}{6}+O(dx^5)$$

Kawabe used this method in his research, and will be the method of choice for this investigation.

Numerical Analysis

Equations 18-21 define a three dimensional phase-space function.[34] To observe this system we will apply the Poincare surface of section method. At a particular energy the restriction y=0 defines a two-dimensional surface (x,p_x) in the phase space.[35] Each time a particular trajectory passes through the surface, i.e. each time it crosses the x,p_x plane, a point is plotted at the position of the intersection.

Kawabe investigated the YMHCM system using the fourth-order Runga-Kutta and the Poincare surface of section method. He showed with H=1000 and the vacuum expectation value of the Higgs field, $v=1$, that as the coupling constant κ is decreased from $\kappa=.9$ the phase space becomes chaotic in regions. At $\kappa\cong.1$ the entire phase

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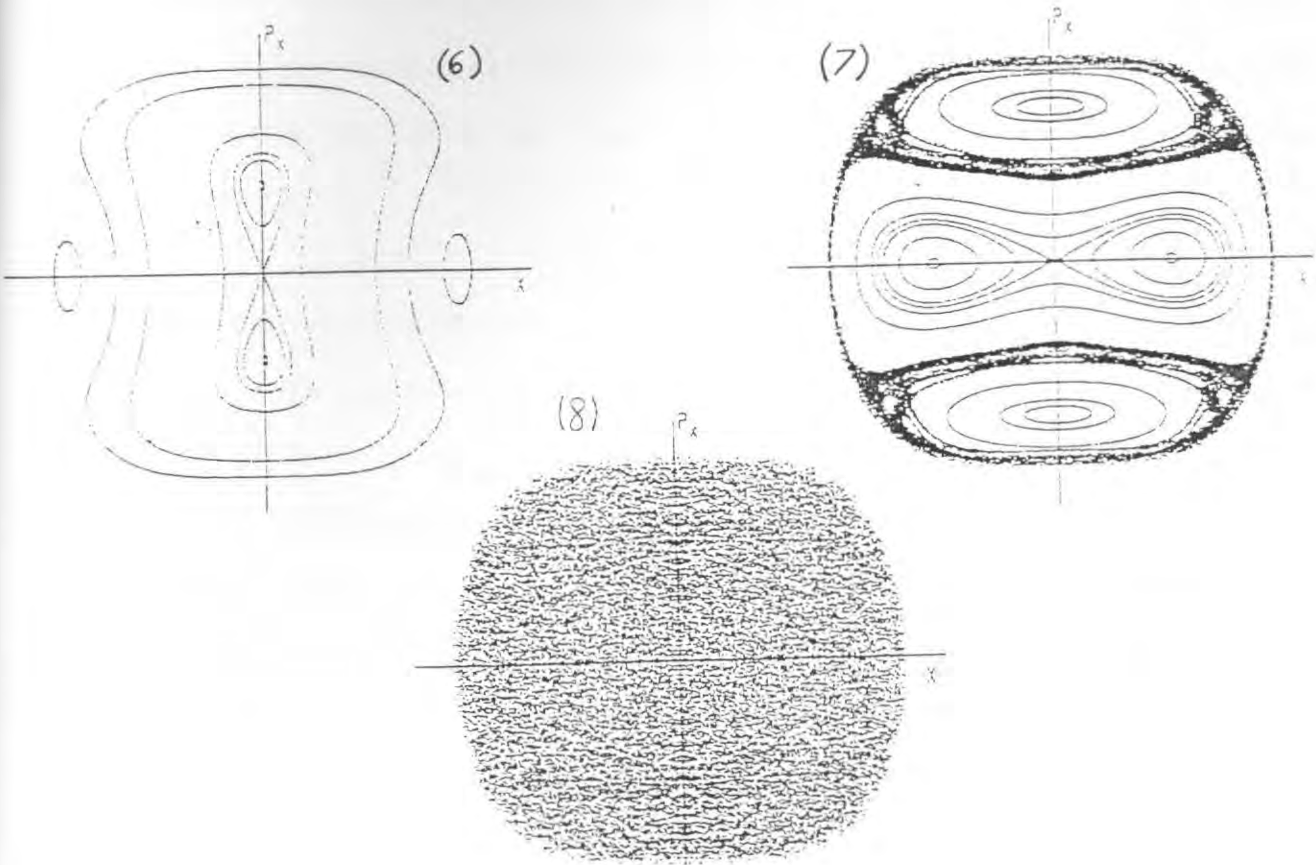
[34]Kawabe, Physics Letters B, 400.

[35] Kawabe, Physics Letters B 400.

space becomes chaotic. See figures 6, 7 and 8.



(Figure 6: κ=0.9. Figure 7: κ=0.4. Figure 8: κ=0.1.)
This is called the critical coupling constant, κ_c. At this level and below the entire phase space is chaotic. Therefore, the YMHCM system exhibits chaos.

Our research focused upon reproducing the results of Kawabe. We utilized a fourth-order Runga-Kutta method. We set H=1000 and used a step size of h=.001. We found that for the values of κ=1.0, 0.9, 0.5, 0.1, 0.01; ν=1.0, 0.9, 0.5, 0.1, 0.01 chaos does not appear. Rather, we reproduced smooth KAM curves like those of figure 6. This is in direct contradiction to the work of Kawabe.

As of the writing of this thesis, the author has been unable to contact Dr. Kawabe regarding these results. This result suggests that the Higgs field has no part in chaos. Therefore the heating of the early universe in the inflation model must be caused by another source.

Conclusions

The Standard Model of the universe has revolutionized particle physics. With the introduction of quantum field theories, theorist began to attempt to unite the forces into one theory. This lead to an attempt to combine the electromagnetic and weak force into an electroweak interaction. This proved to be quite a problem with many difficulties arising. Peter Higgs solved the most vexing problem with the introduction of the Higgs field and a new particle to the Standard Model, the Higgs boson. The equations of this electroweak field coupled to the Higgs field yields the Yang-Mills-Higgs Classical Mechanics equations. These equations were integrated via the fourth order Runge-Kutta method. Kawabe showed that this system exhibits an ordered to chaos transition with respect to the coupling constant. Our research of these same equations using the same numerical method revealed that there are no values for which chaos appears. This would suggest that there is a different method by which the inflationary model of the early universe was heated.

Appendix

This is a copy of the program which was used to integrate the YMHCM equations. It is written in ANSI C, with graphics output to a VT-340 terminal.

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <time.h>
#include "nr.h"
#include "graphics.h"
#include "curses.h"

#define N 4
#define MOD %
#define NR_END 1
#define FREE_ARG char*

float h=.001;
float E, kappa, nu;

void derivs(float x, float y[], float dydx[]);
void rk4(float y[], float dydx[], int n, float x, float h, float
        yout[], void (*derivs)(float, float [], float []));
void nrerror(char error_text[]);
float *vector(long nl, long nh);
void free_vector(float *v, long nl, long nh);


void main(void)
{
   float j, k, *eq, *deqdt, t, tmax = 2.0, xo, xdoto, yo, ydoto, W,
        Etemp;
   int xscr, yscr;

   cls();
   move_curs(27,8); printf("**********************************");
   move_curs(27,9);  printf("*    YMHCM Graphing Program    *");
   move_curs(27,10);  printf("**********************************");
   move_curs(27,12);  printf("Please input the following.");
   move_curs(27,13);  printf("Energy of the system?      ");
   scanf("%f",&E);
   move_curs(27,14);  printf("Coupling constant?          ");
   scanf("%f",&kappa);
   move_curs(27,15);  printf("Higgs value?                ");
   scanf("%f",&nu);
   cls();

   set_graphics_mode(1);
   turn_cursor_off();
```

```
    srand(time(NULL));
    eq = vector(1,N);
    deqdt = vector(1,N);

    for(j=-6.0; j<=6.0; j+=.1)
    for(k=-6.0; k<=6.0; k+=.1)
    {
       xo = ((float)j);
       ydoto=((float)k);
       xdoto = sqrt(2*E - 1.5*pow(xo,4) - pow(ydoto,2));
       yo = 0.0;

       eq[1] = xo;
       eq[2] = yo;
       eq[3] = xdoto;
       eq[4] = ydoto;

       derivs(0.0, eq, deqdt);

       for(t=0.0; t<tmax; t+=h)
       {
       rk4(eq, deqdt, N, t, h, eq, derivs);
       xscr = 400 + 20*eq[1];
       yscr = 225 - 2*eq[3];
       if((eq[2]<=0.0001) && (eq[2]>=-0.0001))
          plot(xscr, yscr, 'B');
       }
    }
    free_vector(eq,1,N);
    free_vector(deqdt,1,N);

    set_graphics_mode(0);
}

void rk4(float y[], float dydx[], int n, float x, float h, float
yout[],
      void (*derivs)(float, float [], float []))
{
      int i;
      float xh,hh,h6,*dym,*dyt,*yt;

      dym=vector(1,n);
      dyt=vector(1,n);
      yt=vector(1,n);
      hh=h*0.5;
      h6=h/6.0;
      xh=x+hh;
      for (i=1;i<=n;i++) yt[i]=y[i]+hh*dydx[i];
      (*derivs)(xh,yt,dyt);
      for (i=1;i<=n;i++) yt[i]=y[i]+hh*dyt[i];
      (*derivs)(xh,yt,dym);
      for (i=1;i<=n;i++) {
```

```
            yt[i]=y[i]+h*dym[i];
            dym[i] += dyt[i];
        }
        (*derivs)(x+h,yt,dyt);
        for (i=1;i<=n;i++)
            yout[i]=y[i]+h6*(dydx[i]+dyt[i]+2.0*dym[i]);
        free_vector(yt,1,n);
        free_vector(dyt,1,n);
        free_vector(dym,1,n);
}

void nrerror(char error_text[])
/* Numerical Recipes standard error handler */
{
        fprintf(stderr,"Numerical Recipes run-time error...\n");
        fprintf(stderr,"%s\n",error_text);
        fprintf(stderr,"...now exiting to system...\n");
        exit(1);
}

float *vector(long nl, long nh)
/* allocate a float vector with subscript range v[nl..nh] */
{
        float *v;

        v=(float *)malloc((size_t) ((nh-nl+1+NR_END)*sizeof(float)));
        if (!v) nrerror("allocation failure in vector()");
        return v-nl+NR_END;
}

void free_vector(float *v, long nl, long nh)
/* free a float vector allocated with vector() */
{
        free((FREE_ARG) (v+nl-NR_END));
}


void derivs(float x, float y[], float dydx[])
{
   dydx[1] = y[3];
   dydx[2] = y[4];
   dydx[3] = -6.0*pow(y[2],2)*y[1] - 3.0*pow(y[1],3);
   dydx[4] = -6.0*pow(y[1],2)*y[2] - 6.0*kappa*pow(y[2],3) +
             kappa*pow(nu,2)*y[2];
}
```

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