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A Paradox Solved (Or 3)

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Why the Interest?

- Introduced at a Young Age
  - Grandfather Paradox and Time Travel
- Researched More Myself
- Great for Passing Time
  - Something to Do
  - No Materials Required
- Good Exercise for the Mind

What Does Math Have to Do With It?

- Stumped for Years
  - Understood the Problem, But Could Never Figure Out a Solution
  - Worked Through Some of Them Thousands of Times
- Calculus I Idea
  - Teacher's Aide in Cal I Class
  - Took the Idea to Other Paradoxes
- The Mathematical Approach
  - Following Sequence and Logic Every Time Was Insanity
    - Of Course It Led to the Impossible Outcome; That Was the Point!
  - Needed to Change My Approach
    - Look at the Paradoxes Through a Mathematical Lens

The Arrow Problem

- At any instant after an arrow has been shot, it is still. The location is not changing.
- The next instant, it must still be in the same spot, as it was not moving in the previous instant.
- Thus, the motion is nonexistent and the arrow could not move through the air.
- This reasoning can be applied to all objects, so it logically follows that motion cannot exist.

The Arrow Solution: Derivatives

- Math Behind Derivatives Not Important for This
  - It's What They *Are* and What They *Do*
- What is a Derivative?
  - Tells the Slope of a Line at Any Given Point
  - Even Useable for Curves and Graphs with Changing Slopes
- How Does This Apply to the Arrow Paradox?
  - Distance Over Time Graphs
    - Instantaneous Speed
- Okay, But Where's The Problem?
  - Language
  - Hidden Division by Zero

The Arrow Solution Part II

- Language
  - Unchanging Location vs Motion
    - Unchanging Location Does NOT Imply Motionlessness
    - Unchanging Location *Over Time* DOES Imply Motionlessness
- Hidden Division By Zero
  - In an Instant, No Time Passes
  - Speed = Distance Divided by Time
  - Thus, Cannot Say 0 Distance Over 0 Time Implies 0 Speed
- Consecutive Instants
  - Some Time Must Pass Between Instants
  - Otherwise, Same Instant
  - Instantaneous Speed Applies to This Time, However Short
- Thus Premise of Arrow Being Motionless is False,

  And the Arrow is Broken.

Achilles and the Tortoise

- There is a race between a tortoise and the legendary hero Achilles.
- The tortoise gets a head start, then the race begins.
- By the time Achilles gets to where the tortoise began, the tortoise will have gone some distance, and still be ahead.
- Again, by the time Achilles gets to the new distance of the tortoise, it will have moved some small distance further.
- This repeats infinitely, and leads to Achilles never catching the tortoise, as it will always have moved slightly further when Achilles reaches its previous spot.

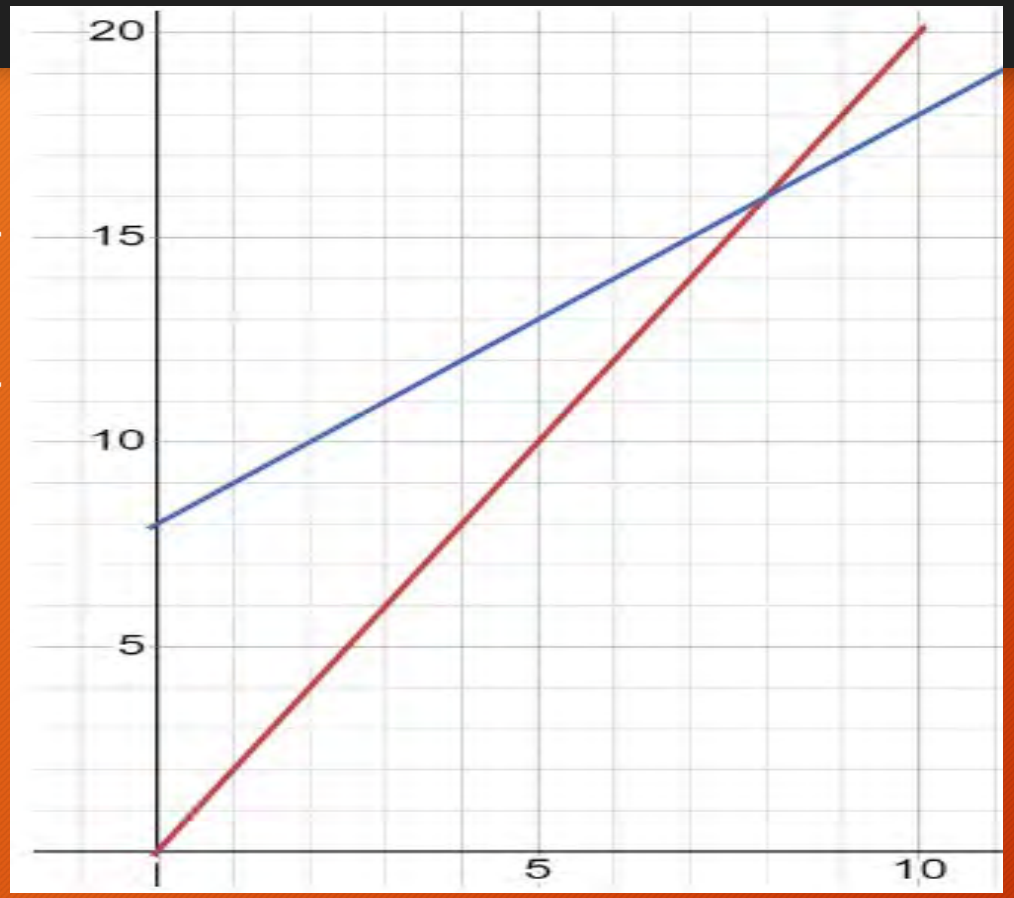
Achilles Solution: Limits

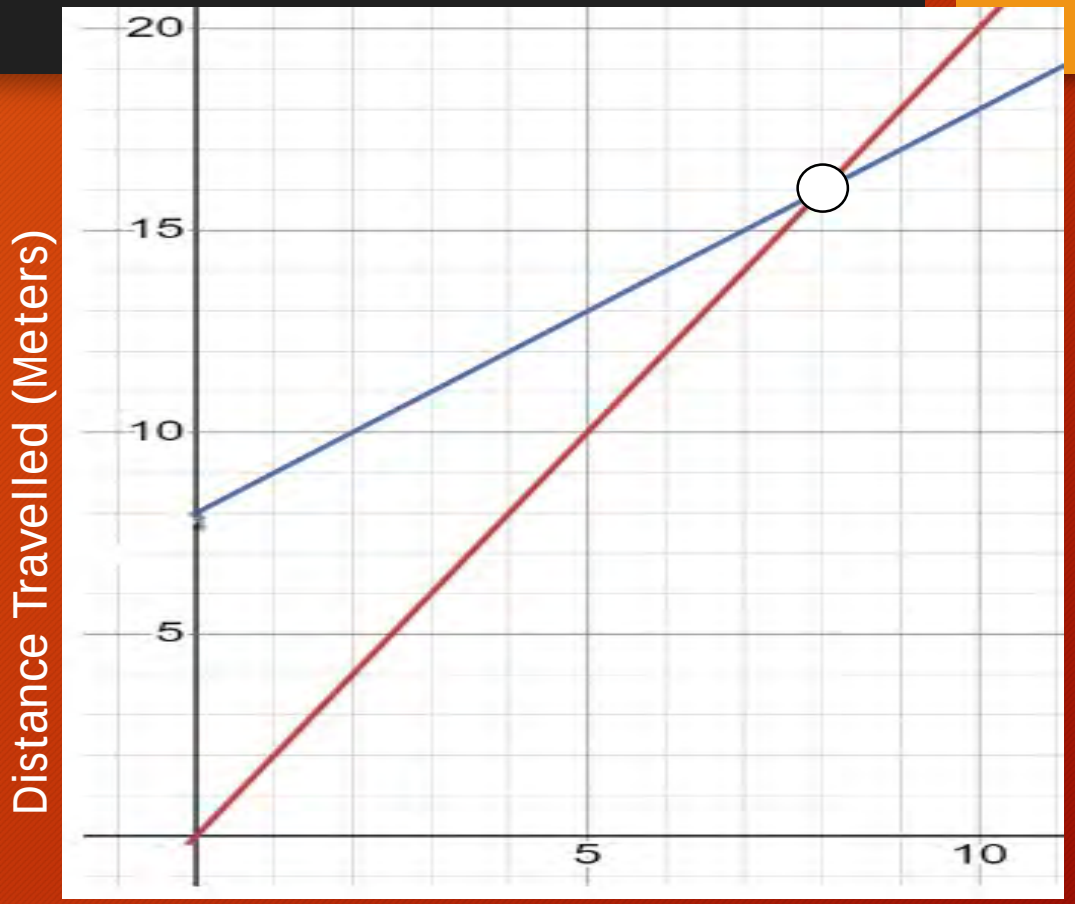
- Obvious Solution: Graphing
  - Shows When Achilles Would Catch the Tortoise
  - Does Not break Down the Paradox or Even Argue the Logic
  - Want to Know Where the Problem Is and How to Beat It
- What is a Limit?
  - Simple Definition: A value an output gets closer and closer to as you approach a specific value of an input
  - Useful as you get close, but not at the moment you reach the point
  - Can Exist at That Point, But May Not
- How Does this Solve the Problem?
  - Hides Limits Through the Language, Gives Implicit Boundaries

Achilles Solution Part II

- The Language
  - The Final Event is Where Achilles Would Catch the Tortoise
  - Limits Maximum Distance and Time
    - Limits Achilles to the Distance Travelled By the Tortoise
    - Takes the Limit of Achilles as He APPROACHES the Tortoise
    - Limits Time to the Point Right Before Achilles Would Catch the Tortoise
- Comparing Graphs
  - Real Graph of the Race
  - Graph Made by Following the Logic
    - Some Limits Can Be Reached, and the Points Do Exist
    - Has a Hole at the Point Where Achilles Catches the Tortoise
- Thus, Achilles Catches the Tortoise and We Have Defeated Achilles.

Race to the Finish Vs Race to the Tortoise





The Dichotomy

- Atalanta wants to take a walk to the end of a path.
- Before reaching the end, she must reach halfway.
- Before that, she must get halfway of halfway.
- This repeats, so she must always go some smaller distance before reaching the next distance.
- Thus, Atalanta can never reach the end of the path, or even begin, as there is so first distance she can travel.

Dichotomy Solution: Limits (But Different)

- Similarities To the Achilles Paradox
  - Breaks Down Distance
  - Hidden Limits
- Differences From the Achilles Paradox
  - Reachable vs Unreachable Limits
  - Maximum vs Minimum Limits
  - Starting Point vs Ending Point
- Thus, a Different Approach Still Using Limits
  - Minimum Limit Cannot Be Reached
    - How the Starting Point Hides This
  - Other Mathematical Steps

Dichotomy Solution Part II

- Minimum Limit
  - Because there is a set starting point, the minimum limit for distance travelled is zero.
    - We know there is some velocity from walking, so Atalanta will have moved from the starting point immediately after time has started.
- Other Mathematical Steps to a Solution
  - Splitting Distance Infinitely
    - Implicitly Splits Time Infinitely
    - Must Take Distance Over an Interval of Time
      - Interval Must Be Greater Than Zero
    - Leads to Limit of Zero Never Being Reached, Problem Solved
- Thus, Atalanta Begins Her Walk And We Finish Ours.